IX. ATTACHED TABLES

1. ATTACHED TABLES

1.1 AQL SAMPLING TABLE (SOURCE: JIS Z 9015)

SAMPLE SIZE CODE LETTERS

Lot size	Special inspection level				Normal inspection level		
	S–1	S–2	S–3	S–4	I	II	III
1- 8	A	A	A	A	A	A	B
9- 15	A	A	A	A	A	B	C
16- 25	A	A	B	B	B	C	D
26- 50	A	B	B	C	C	D	E
51- 90	B	B	C	C	C	E	F
91- 150	B	B	C	D	D	F	G
151- 280	B	C	D	E	E	G	H
281- 500	B	C	D	E	F	H	J
501- 1200	C	C	E	F	G	J	K
1201- 3200	C	D	E	G	H	K	L
3201- 10000	C	D	F	G	J	L	M
10001- 35000	C	D	F	H	K	M	N
35001-150000	D	E	G	J	L	N	P
150001-500000	D	E	G	J	M	P	Q
500001 up	D	E	H	K	N	Q	R

Specify suitable AQLs selecting from 16 levels in a range from 0.010 to 10 for inspections using percent defective (%);
or 26 levels from 0.010 to 1,000 for inspections using defectives per 100units. Unless otherwise specified, use Inspection Standard II.

AQL SAMPLING TABLE (Continued-I)

SINGLE SAMPLING PLANS FOR NORMAL INSPECTION (MASTER TABLE)

Acceptable quality level (AQL) (Normal inspection)

Sample size code letter	Sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ : Use first sampling plan below arrow. If sample size equals or exceeds lot size do 100% inspection.
↑ : Use first sampling plan above arrow.
Ac : Acceptance number
Re : Rejection numbe

AQL SAMPLING TABLE (Continued-II)

SINGLE SAMPLING PLANS FOR SEVERE INSPECTION (MASTER TABLE)

Sample size code letter	Sample size	Acceptable quality level (AQL) (Severe Inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150			1 2																							

↓ : Use first sampling plan below arrow. If sample size equals or exceeds lot size do 100% inspection.
↑ : Use first sampling plan above arrow.
Ac : Acceptance number
Re : Rejection numbe

AQL SAMPLING TABLE (Continued-III)

SINGLE SAMPLING PLANS FOR SLACK INSPECTION (MASTER TABLE)

Acceptable quality level (AQL) (Slack inspection) †

Sample size code letter	Sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	↓	↓	2 3	2 4	3 5	5 6	7 8	10 11	14 15	21 22	30 31
B	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 2	↑	↓	1 3	2 3	2 4	3 5	5 6	7 8	10 11	14 15	21 22	30 31
C	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑
D	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑
E	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	14 17	21 24	↑	↑	↑
F	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑
G	13	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑
H	20	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑
J	32	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	50	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	80	↓	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	125	↓	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	200	↓	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	315	↓	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	500	0 1	↑	↓	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	800	↑	↑	1 2	1 3	2 4	2 5	3 6	5 8	7 10	10 13	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ : Use first sampling plan below arrow. If sample size equals or exceeds lot size do 100% inspection.
↑ : Use first sampling plan above arrow.
Ac : Acceptance number
Re : Rejection numbe
† : The lot is accepted when failures are greater than Ac but less than Re. Subsequent lots are, however, subject to normal inspection.

1.2 LTPD SAMPLING TABLE (SOURCE: MIL-S-19500, SAMPLING INSPECTION TABLES)

The minimum required sample size to obtain a 90% confidence level that a lot of the same failure rate as that of the specified LTPD

Lot tolerance percent defective (LTPD) or λ	50	30	20	15	10	7	5	3
Number of failures allowed (C) (r = C + 1) Minimum sample size (×1,000 for elements required in life test × time)								
0	5 (1.03)	8 (0.64)	11 (0.46)	15 (0.34)	22 (0.23)	32 (0.16)	45 (0.11)	76 (0.07)
1	8 (4.4)	13 (2.7)	18 (2.0)	25 (1.4)	38 (0.94)	55 (0.65)	77 (0.46)	129 (0.28)
2	11 (7.4)	18 (4.5)	25 (3.4)	34 (2.24)	52 (1.6)	75 (1.1)	105 (0.78)	176 (0.47)
3	13 (10.5)	22 (6.2)	32 (4.4)	43 (3.2)	65 (2.1)	94 (1.5)	132 (1.0)	221 (0.62)
4	16 (12.3)	27 (7.3)	38 (5.3)	52 (3.9)	78 (2.6)	113 (1.8)	158 (1.3)	265 (0.75)
5	19 (13.8)	31 (8.4)	45 (6.0)	60 (4.4)	91 (2.9)	131 (2.0)	184 (1.4)	308 (0.85)
6	21 (15.6)	35 (9.4)	51 (6.6)	68 (4.9)	104 (3.2)	149 (2.2)	209 (1.6)	349 (0.94)
7	24 (16.6)	39 (10.2)	57 (7.2)	77 (5.3)	116 (3.5)	166 (2.4)	234 (1.7)	390 (1.0)
8	26 (18.1)	43 (10.9)	63 (7.7)	85 (5.6)	128 (3.7)	184 (2.6)	258 (1.8)	431 (1.1)
9	28 (19.4)	47 (11.5)	69 (8.1)	93 (6.0)	140 (3.9)	201 (2.7)	282 (1.9)	471 (1.2)
10	31 (19.9)	51 (12.1)	75 (8.4)	100 (6.3)	152 (4.1)	218 (2.9)	306 (2.0)	511 (1.2)
11	33 (21.0)	54 (12.8)	83 (8.3)	111 (6.2)	166 (4.2)	238 (2.9)	332 (2.1)	555 (1.2)
12	36 (21.4)	59 (13.0)	89 (8.6)	119 (6.5)	178 (4.3)	254 (3.0)	356 (2.2)	594 (1.3)
13	38 (22.3)	63 (13.4)	95 (8.9)	126 (6.7)	190 (4.5)	271 (3.1)	379 (2.26)	632 (1.3)
14	40 (23.1)	67 (13.8)	101 (9.2)	134 (6.9)	201 (4.6)	288 (3.2)	403 (2.3)	672 (1.4)
15	43 (23.3)	71 (14.1)	107 (9.4)	142 (7.1)	213 (4.7)	305 (3.3)	426 (2.36)	711 (1.41)
16	45 (24.1)	74 (14.6)	112 (9.7)	150 (7.2)	225 (4.8)	321 (3.37)	450 (2.41)	750 (1.44)
17	47 (24.7)	79 (14.7)	118 (9.86)	158 (7.36)	236 (4.93)	338 (3.44)	473 (2.46)	788 (1.48)
18	50 (24.9)	83 (15.0)	124 (10.0)	165 (7.54)	248 (5.02)	354 (3.51)	496 (2.51)	826 (1.51)
19	52 (25.5)	86 (15.4)	130 (10.2)	173 (7.76)	259 (5.12)	370 (3.58)	518 (2.56)	864 (1.53)
20	54 (26.1)	90 (15.6)	135 (10.4)	180 (7.82)	271 (5.19)	386 (3.65)	541 (2.60)	902 (1.56)
25	65 (27.0)	109 (16.1)	163 (10.8)	217 (8.08)	326 (5.38)	466 (3.76)	652 (2.69)	1,086 (1.61)

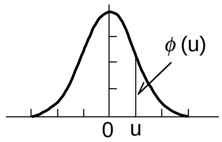
Notes 1. The number of samples is determined according to the limits of the Poisson binomial distribution indexes.
2. Values in parentheses indicate the minimum quality required to have 19 out of 20 lots pass (on average). This approximates AQL values.

will not be passed. (Single sampling)

2	1.5	1	0.7	0.5	0.3	0.2	0.15	0.1
116 (0.04)	153 (0.03)	231 (0.02)	328 (0.02)	461 (0.01)	767 (0.007)	1152 (0.005)	1534 (0.003)	2303 (0.002)
195 (0.18)	258 (0.14)	390 (0.09)	555 (0.06)	778 (0.045)	1296 (0.027)	1946 (0.018)	2592 (0.013)	3891 (0.009)
266 (0.31)	354 (0.23)	533 (0.15)	759 (0.11)	1065 (0.08)	1773 (0.045)	2662 (0.031)	3547 (0.022)	5323 (0.015)
333 (0.41)	444 (0.32)	668 (0.20)	953 (0.14)	1337 (0.10)	2226 (0.062)	3341 (0.041)	4452 (0.031)	6681 (0.018)
398 (0.51)	531 (0.37)	798 (0.52)	1140 (0.17)	1599 (0.12)	2663 (0.074)	3997 (0.049)	5327 (0.037)	7994 (0.025)
462 (0.57)	617 (0.42)	927 (0.28)	1323 (0.20)	1855 (0.14)	3090 (0.085)	4638 (0.056)	6181 (0.042)	9275 (0.028)
528 (0.62)	700 (0.47)	1054 (0.31)	1503 (0.22)	2107 (0.155)	3509 (0.093)	5267 (0.052)	7019 (0.074)	10533 (0.031)
589 (0.67)	783 (0.51)	1178 (0.34)	1680 (0.24)	2355 (0.17)	3922 (0.101)	5886 (0.067)	7845 (0.051)	11771 (0.034)
648 (0.72)	864 (0.54)	1300 (0.36)	1854 (0.25)	2599 (0.18)	4329 (0.108)	6498 (0.072)	8660 (0.054)	12995 (0.036)
709 (0.77)	945 (0.58)	1421 (0.38)	2027 (0.27)	2842 (0.19)	4733 (0.114)	7103 (0.077)	9468 (0.057)	14206 (0.038)
770 (0.80)	1025 (0.60)	1541 (0.40)	2199 (0.28)	3082 (0.20)	5133 (0.120)	7704 (0.080)	10268 (0.060)	15407 (0.040)
832 (0.83)	1109 (0.62)	1664 (0.42)	2378 (0.29)	3323 (0.21)	5546 (0.12)	8319 (0.083)	11092 (0.062)	16638 (0.042)
890 (0.86)	1187 (0.65)	1781 (0.43)	2544 (0.3)	3562 (0.22)	5936 (0.13)	8904 (0.086)	11872 (0.065)	17808 (0.043)
948 (0.89)	1264 (0.67)	1896 (0.44)	2709 (0.31)	3793 (0.22)	6321 (0.134)	9482 (0.089)	12643 (0.067)	18964 (0.045)
1007 (0.92)	1343 (0.69)	2015 (0.46)	2878 (0.32)	4029 (0.23)	6716 (0.138)	10073 (0.092)	13431 (0.069)	20146 (0.046)
1066 (0.94)	1422 (0.71)	2133 (0.47)	3046 (0.33)	4265 (0.235)	7108 (0.141)	10662 (0.094)	14216 (0.070)	21324 (0.047)
1124 (0.96)	1499 (0.72)	2249 (0.48)	3212 (0.337)	4497 (0.241)	7496 (0.144)	11244 (0.096)	14992 (0.072)	22487 (0.048)
1182 (0.98)	1576 (0.74)	2364 (0.49)	3377 (0.344)	4728 (0.246)	7880 (0.148)	11819 (0.098)	15759 (0.074)	23639 (0.049)
1239 (1.0)	1652 (0.75)	2478 (0.50)	3540 (0.351)	4956 (0.251)	8260 (0.151)	12390 (0.100)	16520 (0.075)	24780 (0.050)
1296 (1.02)	1728 (0.77)	2591 (0.52)	3702 (0.358)	5183 (0.256)	8638 (0.153)	12957 (0.102)	17276 (0.077)	25914 (0.051)
1353 (1.04)	1803 (0.78)	2705 (0.52)	3864 (0.364)	5410 (0.260)	9017 (0.156)	13526 (0.104)	18034 (0.078)	27051 (0.052)
1629 (1.08)	2173 (0.807)	3259 (0.538)	4656 (0.376)	6518 (0.259)	10863 (0.161)	16295 (0.108)	21726 (0.081)	32589 (0.054)

1.3 PROBABILITY DENSITY OF NORMAL DISTRIBUTION

$$\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$



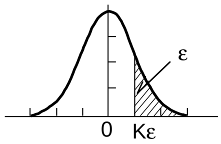
u	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.39894	.39892	.39886	.39876	.39862	.39844	.39822	.39797	.39767	.39733
.1	.39695	.39654	.39608	.39559	.39505	.39448	.39387	.39322	.39253	.39181
.2	.39104	.39024	.38940	.38853	.38762	.38667	.38568	.38466	.38361	.38251
.3	.38139	.38023	.37903	.37780	.37654	.37524	.37391	.37255	.37115	.36973
.4	.36827	.36678	.36526	.36371	.36213	.36053	.35889	.35723	.35553	.35381
.5	.35207	.35029	.34849	.34667	.34482	.34294	.34105	.33912	.33718	.33521
.6	.33322	.33121	.32918	.32713	.32506	.32297	.32086	.31874	.31659	.31443
.7	.31225	.31006	.30785	.30563	.30339	.30114	.29887	.29659	.29431	.29200
.8	.28969	.28737	.28504	.28269	.28034	.27798	.27562	.27324	.27086	.26848
.9	.26609	.26369	.26129	.25888	.25647	.25406	.25164	.24923	.24681	.24439
1.0	.24197	.23955	.23713	.23471	.23230	.22988	.22747	.22506	.22265	.22025
1.1	.21785	.21546	.21307	.21069	.20831	.20594	.20357	.20121	.19886	.19652
1.2	.19419	.19186	.18954	.18724	.18494	.18265	.18037	.17810	.17585	.17360
1.3	.17137	.16915	.16694	.16474	.16256	.16038	.15822	.15608	.15395	.15183
1.4	.14973	.14764	.14556	.14350	.14146	.13943	.13742	.13542	.13344	.13147
1.5	.12952	.12758	.12566	.12376	.12188	.12001	.11816	.11632	.11450	.11270
1.6	.11092	.10915	.10741	.10567	.10396	.10226	.10059	.098925	.097282	.095657
1.7	.094049	.092459	.090887	.089333	.087796	.086277	.084776	.083293	.081828	.080380
1.8	.078950	.077538	.076143	.074766	.073407	.072065	.070740	.069433	.068144	.066871
1.9	.065616	.064378	.063157	.061952	.060765	.059595	.058441	.057304	.056183	.055079
2.0	.053991	.052919	.051864	.050824	.049800	.048792	.047800	.046823	.045861	.044915
2.1	.043984	.043067	.042166	.041280	.040408	.039550	.038707	.037878	.037063	.036262
2.2	.035475	.034701	.033941	.033194	.032460	.031740	.031032	.030337	.029655	.028985
2.3	.028327	.027682	.027048	.026426	.025817	.025218	.024631	.024056	.023491	.022937
2.4	.022395	.021862	.021341	.020829	.020328	.019837	.019356	.018885	.018423	.017971
2.5	.017528	.017095	.016670	.016254	.015848	.015449	.015060	.014678	.014305	.013940
2.6	.013583	.013234	.012892	.012558	.012232	.011912	.011600	.011295	.010997	.010706
2.7	.010421	.010143	$.0^{2}98712$	$.0^{2}96058$	$.0^{2}93466$	$.0^{2}90936$	$.0^{2}88465$	$.0^{2}86052$	$.0^{2}83697$	$.0^{2}81398$
2.8	$.0^{2}79155$	$.0^{2}76965$	$.0^{2}74829$	$.0^{2}72744$	$.0^{2}70711$	$.0^{2}68728$	$.0^{2}66793$	$.0^{2}64907$	$.0^{2}63067$	$.0^{2}61274$
2.9	$.0^{2}59525$	$.0^{2}57821$	$.0^{2}56160$	$.0^{2}54541$	$.0^{2}52963$	$.0^{2}51426$	$.0^{2}49929$	$.0^{2}48470$	$.0^{2}47050$	$.0^{2}45666$
3.0	$.0^{2}44318$	$.0^{2}43007$	$.0^{2}41729$	$.0^{2}40486$	$.0^{2}39276$	$.0^{2}38098$	$.0^{2}36951$	$.0^{2}35836$	$.0^{2}34751$	$.0^{2}33695$
3.1	$.0^{2}32668$	$.0^{2}31669$	$.0^{2}30698$	$.0^{2}29754$	$.0^{2}28835$	$.0^{2}27943$	$.0^{2}27075$	$.0^{2}26231$	$.0^{2}25412$	$.0^{2}24615$
3.2	$.0^{2}23841$	$.0^{2}23089$	$.0^{2}22358$	$.0^{2}21649$	$.0^{2}20960$	$.0^{2}20290$	$.0^{2}19641$	$.0^{2}19010$	$.0^{2}18397$	$.0^{2}17803$
3.3	$.0^{2}17226$	$.0^{2}16666$	$.0^{2}16122$	$.0^{2}15595$	$.0^{2}15084$	$.0^{2}14587$	$.0^{2}14106$	$.0^{2}13639$	$.0^{2}13187$	$.0^{2}12748$
3.4	$.0^{2}12322$	$.0^{2}11910$	$.0^{2}11510$	$.0^{2}11122$	$.0^{2}10747$	$.0^{2}10383$	$.0^{2}10030$	$.0^{3}96886$	$.0^{3}93577$	$.0^{3}90372$
3.5	$.0^{3}87268$	$.0^{3}84263$	$.0^{3}81352$	$.0^{3}78534$	$.0^{3}75807$	$.0^{3}73166$	$.0^{3}70611$	$.0^{3}68138$	$.0^{3}65745$	$.0^{3}63430$
3.6	$.0^{3}61190$	$.0^{3}59024$	$.0^{3}56928$	$.0^{3}54901$	$.0^{3}52941$	$.0^{3}51046$	$.0^{3}49214$	$.0^{3}47443$	$.0^{3}45731$	$.0^{3}44077$
3.7	$.0^{3}42478$	$.0^{3}40933$	$.0^{3}39440$	$.0^{3}37998$	$.0^{3}36605$	$.0^{3}35260$	$.0^{3}33960$	$.0^{3}32705$	$.0^{3}31494$	$.0^{3}30324$
3.8	$.0^{3}29195$	$.0^{3}28105$	$.0^{3}27053$	$.0^{3}26037$	$.0^{3}25058$	$.0^{3}24113$	$.0^{3}23201$	$.0^{3}22321$	$.0^{3}21473$	$.0^{3}20655$
3.9	$.0^{3}19866$	$.0^{3}19105$	$.0^{3}18371$	$.0^{3}17664$	$.0^{3}16983$	$.0^{3}16326$	$.0^{3}15693$	$.0^{3}15083$	$.0^{3}14495$	$.0^{3}13928$
4.0	$.0^{3}13383$	$.0^{3}12858$	$.0^{3}12352$	$.0^{3}11864$	$.0^{3}11395$	$.0^{3}10943$	$.0^{3}10509$	$.0^{3}10090$	$.0^{4}96870$	$.0^{4}92993$
4.1	$.0^{4}89262$	$.0^{4}85672$	$.0^{4}82218$	$.0^{4}78895$	$.0^{4}75700$	$.0^{4}72626$	$.0^{4}69670$	$.0^{4}66828$	$.0^{4}64095$	$.0^{4}61468$
4.2	$.0^{4}58943$	$.0^{4}56516$	$.0^{4}54183$	$.0^{4}51942$	$.0^{4}49788$	$.0^{4}47719$	$.0^{4}45731$	$.0^{4}43821$	$.0^{4}41988$	$.0^{4}40226$
4.3	$.0^{4}38535$	$.0^{4}36911$	$.0^{4}35353$	$.0^{4}33856$	$.0^{4}32420$	$.0^{4}31041$	$.0^{4}29719$	$.0^{4}28499$	$.0^{4}27231$	$.0^{4}26063$
4.4	$.0^{4}24942$	$.0^{4}23868$	$.0^{4}22837$	$.0^{4}21848$	$.0^{4}20900$	$.0^{4}19992$	$.0^{4}19121$	$.0^{4}18286$	$.0^{4}17486$	$.0^{4}16719$
4.5	$.0^{4}15984$	$.0^{4}15280$	$.0^{4}14605$	$.0^{4}13959$	$.0^{4}13340$	$.0^{4}12747$	$.0^{4}12180$	$.0^{4}11636$	$.0^{4}11116$	$.0^{4}10618$
4.6	$.0^{4}10141$	$.0^{5}96845$	$.0^{5}92477$	$.0^{5}88297$	$.0^{5}84298$	$.0^{5}80472$	$.0^{5}76812$	$.0^{5}73311$	$.0^{5}69962$	$.0^{5}66760$
4.7	$.0^{5}63698$	$.0^{5}60771$	$.0^{5}57972$	$.0^{5}55296$	$.0^{5}52739$	$.0^{5}50295$	$.0^{5}47960$	$.0^{5}45728$	$.0^{5}43596$	$.0^{5}41559$
4.8	$.0^{5}39613$	$.0^{5}37755$	$.0^{5}35980$	$.0^{5}34285$	$.0^{5}32667$	$.0^{5}31122$	$.0^{5}29647$	$.0^{5}28239$	$.0^{5}26895$	$.0^{5}25613$
4.9	$.0^{5}24390$	$.0^{5}23222$	$.0^{5}22108$	$.0^{5}21046$	$.0^{5}20033$	$.0^{5}19066$	$.0^{5}18144$	$.0^{5}17265$	$.0^{5}16428$	$.0^{5}15629$

The left-hand and top values are used to identify the value of the deviation u. The table value listed at the intersection of these two values is the probability density $\phi(u)$ at this value of u.

Example: For u=2.96, find the value located at the intersection of 2.9 on the left and .06 on the top. This value, $.0^{2}49929$ (=0.0049929) is the value of $\phi(u)$ for u = 2.96.

1.4 UPPER PROBABILITY OF NORMAL DISTRIBUTION

$$\varepsilon\,(K\varepsilon) : \varepsilon = \int_{K\varepsilon}^{\infty} \phi\,(u)\,du$$



Kε	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	28096	.27760
.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.098525
1.3	.096800	.095098	.093418	.091759	.090123	.088508	.086915	.085343	.083793	.082264
1.4	.080757	.079270	.077804	.076359	.074934	.073529	.072145	.070781	.069437	.068112
1.5	.066807	.065522	.064255	.063008	.061780	.060571	.059380	.058208	.057053	.055917
1.6	.054799	.053699	.052616	.051551	.050503	.049471	.048457	.047460	.046479	.045514
1.7	.044565	.043683	.042716	.041815	.040930	.040059	.039204	.038364	.037538	.036727
1.8	.035930	.035148	.034380	.033625	.032884	.032157	.031443	.030742	.030054	.029379
1.9	.028717	.028067	.027429	.026803	.026190	.025588	.024998	.024419	.023852	.023295
2.0	.022750	.022216	.021692	.021178	.020675	.020182	.019699	.019226	.018763	.018309
2.1	.017864	.017429	.017003	.016586	.016177	.015778	.015386	.015003	.014629	.014262
2.2	.013903	.013553	.013209	.012874	.012545	.012224	.011911	.011604	.011304	.011011
2.3	.010724	.010444	.010170	$.0^{2}99031$	$.0^{2}96419$	$.0^{2}93867$	$.0^{2}91375$	$.0^{2}88940$	$.0^{2}86563$	$.0^{2}84242$
2.4	$.0^{2}81975$	$.0^{2}79763$	$.0^{2}77603$	$.0^{2}75494$	$.0^{2}73436$	$.0^{2}71428$	$.0^{2}99469$	$.0^{2}67557$	$.0^{2}65691$	$.0^{2}63872$
2.5	$.0^{2}62097$	$.0^{2}60366$	$.0^{2}58677$	$.0^{2}57031$	$.0^{2}55426$	$.0^{2}53861$	$.0^{2}52336$	$.0^{2}50849$	$.0^{2}49400$	$.0^{2}47988$
2.6	$.0^{2}46612$	$.0^{2}45271$	$.0^{2}43965$	$.0^{2}42692$	$.0^{2}41453$	$.0^{2}40246$	$.0^{2}39070$	$.0^{2}37926$	$.0^{2}36811$	$.0^{2}35726$
2.7	$.0^{2}34670$	$.0^{2}33642$	$.0^{2}32641$	$.0^{2}31667$	$.0^{2}30720$	$.0^{2}29798$	$.0^{2}28901$	$.0^{2}28028$	$.0^{2}27179$	$.0^{2}26354$
2.8	$.0^{2}25551$	$.0^{2}24771$	$.0^{2}24012$	$.0^{2}23274$	$.0^{2}22557$	$.0^{2}21860$	$.0^{2}21182$	$.0^{2}20524$	$.0^{2}19884$	$.0^{2}19262$
2.9	$.0^{2}18658$	$.0^{2}18071$	$.0^{2}17502$	$.0^{2}16948$	$.0^{2}16411$	$.0^{2}15889$	$.0^{2}15382$	$.0^{2}14890$	$.0^{2}14412$	$.0^{2}13949$
3.0	$.0^{2}13499$	$.0^{2}13062$	$.0^{2}12639$	$.0^{2}12228$	$.0^{2}11829$	$.0^{2}11442$	$.0^{2}11067$	$.0^{2}10703$	$.0^{2}10350$	$.0^{2}10008$
3.1	$.0^{3}96760$	$.0^{3}93544$	$.0^{3}90426$	$.0^{3}87403$	$.0^{3}84474$	$.0^{3}81635$	$.0^{3}78885$	$.0^{3}76219$	$.0^{3}73638$	$.0^{3}71136$
3.2	$.0^{3}68714$	$.0^{3}66367$	$.0^{3}64095$	$.0^{3}61895$	$.0^{3}59765$	$.0^{3}57703$	$.0^{3}55706$	$.0^{3}53774$	$.0^{3}51904$	$.0^{3}50094$
3.3	$.0^{3}48342$	$.0^{3}46648$	$.0^{3}45009$	$.0^{3}43423$	$.0^{3}41889$	$.0^{3}40406$	$.0^{3}38971$	$.0^{3}37584$	$.0^{3}36243$	$.0^{3}34946$
3.4	$.0^{3}33693$	$.0^{3}32481$	$.0^{3}31311$	$.0^{3}30179$	$.0^{3}29086$	$.0^{3}28029$	$.0^{3}27009$	$.0^{3}26023$	$.0^{3}25071$	$.0^{3}24151$
3.5	$.0^{3}23263$	$.0^{3}22405$	$.0^{3}21577$	$.0^{3}20778$	$.0^{3}20006$	$.0^{3}19262$	$.0^{3}18543$	$.0^{3}17849$	$.0^{3}17180$	$.0^{3}16534$
3.6	$.0^{3}15911$	$.0^{3}15310$	$.0^{3}14730$	$.0^{3}14171$	$.0^{3}13632$	$.0^{3}13112$	$.0^{3}12611$	$.0^{3}12128$	$.0^{3}11662$	$.0^{3}11213$
3.7	$.0^{3}10780$	$.0^{3}10363$	$.0^{4}99611$	$.0^{4}95740$	$.0^{4}92010$	$.0^{4}88417$	$.0^{4}84957$	$.0^{4}81624$	$.0^{4}78414$	$.0^{4}75324$
3.8	$.0^{4}72348$	$.0^{4}69483$	$.0^{4}66726$	$.0^{4}64072$	$.0^{4}61517$	$.0^{4}59059$	$.0^{4}56694$	$.0^{4}54418$	$.0^{4}52228$	$.0^{4}50122$
3.9	$.0^{4}48096$	$.0^{4}46148$	$.0^{4}44274$	$.0^{4}42473$	$.0^{4}40741$	$.0^{4}39076$	$.0^{4}37475$	$.0^{4}35936$	$.0^{4}34458$	$.0^{4}33037$
4.0	$.0^{4}31671$	$.0^{4}30359$	$.0^{4}29099$	$.0^{4}27888$	$.0^{4}26726$	$.0^{4}25609$	$.0^{4}24536$	$.0^{4}23507$	$.0^{4}22518$	$.0^{4}21569$
4.1	$.0^{4}20658$	$.0^{4}19783$	$.0^{4}18944$	$.0^{4}18138$	$.0^{4}17365$	$.0^{4}16624$	$.0^{4}15912$	$.0^{4}15230$	$.0^{4}14575$	$.0^{4}13948$
4.2	$.0^{4}13346$	$.0^{4}12769$	$.0^{4}12215$	$.0^{4}11685$	$.0^{4}11176$	$.0^{4}10689$	$.0^{4}10221$	$.0^{5}97736$	$.0^{5}93447$	$.0^{5}89337$
4.3	$.0^{5}85399$	$.0^{5}81627$	$.0^{5}78015$	$.0^{5}74555$	$.0^{5}71241$	$.0^{5}68069$	$.0^{5}65031$	$.0^{5}62123$	$.0^{5}59340$	$.0^{5}56675$
4.4	$.0^{5}54125$	$.0^{5}51685$	$.0^{5}49350$	$.0^{5}47117$	$.0^{5}44979$	$.0^{5}42935$	$.0^{5}40980$	$.0^{5}39110$	$.0^{5}37322$	$.0^{5}35612$
4.5	$.0^{5}33977$	$.0^{5}32414$	$.0^{5}30920$	$.0^{5}29492$	$.0^{5}28127$	$.0^{5}26823$	$.0^{5}25577$	$.0^{5}24386$	$.0^{5}23249$	$.0^{5}22162$
4.6	$.0^{5}21125$	$.0^{5}20133$	$.0^{5}19187$	$.0^{5}18283$	$.0^{5}17420$	$.0^{5}16597$	$.0^{5}15810$	$.0^{5}15060$	$.0^{5}14344$	$.0^{5}13660$
4.7	$.0^{5}13008$	$.0^{5}12386$	$.0^{5}11792$	$.0^{5}11226$	$.0^{5}10686$	$.0^{5}10171$	$.0^{6}96796$	$.0^{6}92113$	$.0^{6}87648$	$.0^{6}83391$
4.8	$.0^{6}79333$	$.0^{6}75465$	$.0^{6}71779$	$.0^{6}68267$	$.0^{6}64920$	$.0^{6}61731$	$.0^{6}58693$	$.0^{6}55799$	$.0^{6}53043$	$.0^{6}50418$
4.9	$.0^{6}47918$	$.0^{6}45538$	$.0^{6}43272$	$.0^{6}41115$	$.0^{6}39061$	$.0^{6}37107$	$.0^{6}35247$	$.0^{6}33476$	$.0^{6}31792$	$.0^{6}30190$

The above table gives the upper probability for a normal distribution for the values of Kε = 0.00 to 4.99.

Example : For Kε = 3.18, find the value located at the intersection of 3.1 on the left and 0.08 on the top.

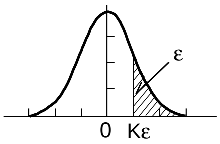
The value $\varepsilon = .0^{3}73638 = 0.00073638$ is the value of the upper probability for Kε = 3.18.

Similarly, for Kε = 1.96, ε = .024998, and for Kε = 2.58, $\varepsilon = .0^{2}49400 = 0.0049400$.

If two-sided probability of distribution is considered, then the above values, respectively, correspond to $2 \cdot \varepsilon = 0.049996 \geq 0.05$ and $0.00988 \fallingdotseq 0.01$

1.5 PERCENT POINTS OF NORMAL DISTRIBUTION

$K\varepsilon\,(\varepsilon) : \varepsilon = \int_{K\varepsilon}^{\infty} \phi\,(u)\,du$



ε	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	∞	3.09023	2.87816	2.74778	2.65207	2.57583	2.51214	2.45726	2.40892	2.36562
.01	2.32635	2.29037	2.25713	2.22621	2.19729	2.17009	2.14441	2.12007	2.09693	2.07485
.02	2.05375	2.03352	2.01409	1.99539	1.97737	1.95996	1.94313	1.92684	1.91104	1.89570
.03	1.88079	1.86630	1.85218	1.83842	1.82501	1.81191	1.79912	1.78661	1.77438	1.76241
.04	1.75069	1.73920	1.72793	1.71689	1.70604	1.69540	1.68494	1.67466	1.66456	1.65463
.05	1.64485	1.63523	1.62576	1.61644	1.60725	1.59819	1.58927	1.58047	1.57179	1.56322
.06	1.55477	1.54643	1.53820	1.53007	1.52204	1.51410	1.50626	1.49851	1.49085	1.48328
.07	1.47579	1.46838	1.46106	1.45381	1.44663	1.43953	1.43250	1.42554	1.41865	1.41183
.08	1.40507	1.39838	1.39174	1.38517	1.37866	1.37220	1.36581	1.35946	1.35317	1.34694
.09	1.34076	1.33462	1.32854	1.32251	1.31652	1.31058	1.30469	1.29884	1.29303	1.28727
.10	1.28155	1.27587	1.27024	1.26464	1.25908	1.25357	1.24808	1.24264	1.23723	1.23186
.11	1.22653	1.22123	1.21596	1.21073	1.20553	1.20036	1.19522	1.19012	1.18504	1.18000
.12	1.17499	1.17000	1.16505	1.16012	1.15522	1.15035	1.14551	1.14069	1.13590	1.13113
.13	1.12639	1.12168	1.11699	1.11232	1.10768	1.10306	1.09847	1.09390	1.08935	1.08482
.14	1.08032	1.07584	1.07138	1.06694	1.06252	1.05812	1.05374	1.04939	1.04505	1.04073
.15	1.03643	1.03215	1.02789	1.02365	1.01943	1.01522	1.01103	1.00686	1.00271	.99858
.16	.99446	.99036	.98627	.98220	.97815	.97411	.97009	.96609	.96210	.95812
.17	.95417	.95022	.94629	.94238	.93848	.93459	.93072	.92686	.92301	.91918
.18	.91537	.91156	.90777	.90399	.90023	.89647	.89273	.88901	.88529	.83159
.19	.87790	.87422	.87055	.86689	.86325	.85962	.85600	.85239	.84879	.84520
.20	.84162	.83805	.83450	.83095	.82742	.82389	.82038	.81687	.81338	.80990
.21	.80642	.80296	.79950	.79606	.79262	.78919	.78577	.78237	.77897	.77557
.22	.77219	.76882	.76546	.76210	.75875	.75542	.75208	.74876	.74545	.74214
.23	.73885	.73556	.73228	.72900	.72574	.72248	.71923	.71599	.71275	.70952
.24	.70630	.70309	.69988	.69668	.69349	.69031	.68713	.68396	.68080	.67764
.25	.67449	.67135	.66821	.66508	.66196	.65884	.65573	.65262	.64952	.64643
.26	.64335	.64027	.63719	.63412	.63106	.62801	.62496	.62191	.61887	.61584
.27	.61281	.60979	.60678	.60376	.60076	.59776	.59477	.59178	.58879	.58581
.28	.58284	.57987	.57691	.57395	.57100	.56805	.56511	.56217	.55924	.55631
.29	.55338	.55047	.54755	.54464	.54174	.53884	.53594	.53305	.53016	.52728
.30	.52440	.52153	.51866	.51579	.51293	.51007	.50722	.50437	.50153	.49869
.31	.49585	.49032	.49019	.48736	.48454	.48173	.47891	.47610	.47330	.47050
.32	.46770	.46490	.46211	.45933	.45654	.45376	.45099	.44821	.44544	.44268
.33	.43991	.43715	.43440	.43164	.42889	.42615	.42340	.42066	.41793	.41519
.34	.41246	.40974	.40701	.40429	.40157	.39886	.39614	.39343	.39073	.38802
.35	.38532	.38262	.37993	.37723	.37454	.37186	.36917	.36649	.36381	.36113
.36	.35846	.35579	.35312	.35045	.34779	.34513	.34247	.33981	.33716	.33450
.37	.33185	.32921	.32656	.32392	.32128	.31864	.31600	.31337	.31074	.30811
.38	.30548	.30286	.30023	.29761	.29499	.29237	.28976	.28715	.28454	.28193
.39	.27932	.27671	.27411	.27151	.26891	.26631	.26371	.26112	.25853	.25594
.40	.25335	.25076	.24817	.24559	.24301	.24043	.23785	.23527	.23269	.23012
.41	.22754	.22497	.22240	.21983	.21727	.21470	.21214	.20957	.20701	.20445
.42	.28189	.19934	.19678	.19422	.19167	.18912	.18657	.18402	.18147	.17892
.43	.17637	.17383	.17128	.16874	.16620	.16366	.16112	.15858	.15604	.15351
.44	.15097	.14843	.14590	.14337	.14084	.13830	.13577	.13324	.13072	.12819
.45	.12566	.12314	.12061	.11809	.11556	.11304	.11052	.10799	.10547	.10295
.46	.10043	.09791	.09540	.09288	.09036	.08784	.08533	.08281	.08030	.07778
.47	.07527	.07276	.07024	.06773	.06522	.06271	.06020	.05768	.05517	.05766
.48	.05015	.04764	.04513	.04263	.04012	.03761	.03510	.03259	.03008	.02758
.49	.02507	.02256	.02005	.01755	.01504	.01253	.01003	.00752	.00501	.00251

The above table gives the value of Kε for the upper probability of normal distribution ε = 0.000 to 0.499.
The Kε value is known as 100 ε percent point.

Example : For ε = 0.200 we find the value at the intersection of .20 on the left side and the .000 on the top.
The value is Kε = .84162. This is referred to as (upper) 20 percent point.
The 2.5% point is represented by the value of ε = 0.025 for which Kε = 1.95996 ≒ 1.96 and the 0.5% point by the value of ε = 0.005, for which Kε = 2.57583 ≒ 2.58.

1.6 POISSON DISTRIBUTION (PROBABILITY)

x	m									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0010	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0002	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x	m									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707
3	.0738	.0867	.0998	.1128	.1255	.1378	.1496	.1607	.1710	.1804
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034
8	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002

x	m									
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1494
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	.1680
5	.0417	.0476	.0538	.0602	.0668	.0735	.0804	.0872	.0940	.1008
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x	m									
	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247	.0224	.0202	.0183
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915	.0850	.0789	.0733
2	.2165	.2087	.2008	.1929	.1850	.0771	.1692	.1615	.1539	.1465
3	.2237	.2226	.2209	.2186	.2158	.2125	.2087	.2046	.2001	.1954
4	.1734	.1781	.1823	.1858	.1888	.1912	.1931	.1944	.1951	.1954
5	.1075	.1140	.1203	.1264	.1322	.1377	.1429	.1477	.1522	.1563
6	.0555	.0608	.0662	.0716	.0711	.0826	.0881	.0936	.0989	.1042
7	.0246	.0278	.0312	.0348	.0385	.0425	.0466	.0508	.0551	.0595
8	.0095	.0111	.0129	.0148	.0169	.0191	.0215	.0241	.0269	.0298
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089	.0102	.0116	.0132
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033	.0039	.0045	.0053
11	.0003	.0004	.0005	.0006	.0007	.0009	.0011	.0013	.0016	.0019
12	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006
13	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x : number of failures detected (percent defective)
m : number of expected failures (number of defective items)

1.6 POISSON DISTRIBUTION (Continued-I)

x	m									
	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	.0166	.0150	.0136	.0123	.0111	.0101	.0091	.0082	.0074	.0067
1	.0679	.0630	.0583	.0540	.0500	.0462	.0427	.0395	.0365	.0337
2	.1393	.1323	.1254	.1188	.1125	.1063	.1005	.0948	.0894	.0842
3	.1904	.1852	.1798	.1743	.1687	.1631	.1574	.1517	.1460	.1404
4	.1951	.1944	.1933	.1917	.1898	.1875	.1849	.1820	.1789	.1755
5	.1600	.1633	.1662	.1687	.1708	.1725	.1738	.1747	.1753	.1755
6	.1093	.1143	.1191	.1237	.1281	.1323	.1362	.1398	.1432	.1462
7	.0640	.0686	.0732	.0778	.0824	.0869	.0914	.0959	.1002	.1044
8	.0328	.0360	.0393	.0428	.0463	.0500	.0537	.0575	.0614	.0653
9	.0150	.0168	.0188	.0209	.0232	.0255	.0280	.0307	.0334	.0363
10	.0061	.0071	.0081	.0092	.0104	.0118	.0132	.0147	.0164	.0181
11	.0023	.0027	.0032	.0037	.0043	.0049	.0056	.0064	.0073	.0082
12	.0008	.0009	.0011	.0014	.0016	.0019	.0022	.0026	.0030	.0034
13	.0002	.0003	.0004	.0005	.0006	.0007	.0008	.0009	.0011	.0013
14	.0001	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005
15	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002

x	m									
	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027	.0025
1	.0311	.0287	.0265	.0244	.0225	.0207	.0191	.0176	.0162	.0149
2	.0793	.0746	.0701	.0659	.0618	.0580	.0544	.0509	.0477	.0446
3	.1348	.1293	.1239	.1185	.1133	.1082	.1033	.0985	.0938	.0892
4	.1719	.1681	.1641	.1600	.1558	.1515	.1472	.1428	.1383	.1339
5	.1753	.1748	.1740	.1728	.1714	.1697	.1678	.1656	.1632	.1606
6	.1490	.1515	.1537	.1555	.1571	.1584	.1594	.1601	.1605	.1606
7	.1086	.1125	.1163	.1200	.1234	.1267	.1298	.1326	.1353	.1377
8	.0692	.0731	.0771	.0810	.0849	.0887	.0925	.0962	.0998	.1033
9	.0392	.0423	.0454	.0486	.0519	.0552	.0586	.0620	.0654	.0688
10	.0200	.0220	.0241	.0262	.0285	.0309	.0334	.0359	.0386	.0413
11	.0093	.0104	.0116	.0129	.0143	.0157	.0173	.0190	.0207	.0225
12	.0039	.0045	.0051	.0058	.0065	.0073	.0082	.0092	.0102	.0113
13	.0015	.0018	.0021	.0024	.0028	.0032	.0036	.0041	.0046	.0052
14	.0006	.0007	.0008	.0009	.0011	.0013	.0015	.0017	.0019	.0022
15	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007	.0008	.0009
16	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003
17	.0001	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001

x	m									
	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	.0022	.0020	.0018	.0017	.0015	.0014	.0012	.0011	.0010	.0009
1	.0137	.0126	.0116	.0106	.0098	.0090	.0082	.0076	.0070	.0064
2	.0417	.0390	.0364	.0340	.0318	.0296	.0276	.0258	.0240	.0223
3	.0848	.0806	.0765	.0726	.0688	.0652	.0617	.0584	.0552	.0521
4	.1294	.1249	.1205	.1162	.1118	.1076	.1034	.0992	.0952	.0912
5	.1579	.1549	.1519	.1487	.1454	.1420	.1385	.1349	.1314	.1277
6	.1605	.1601	.1595	.1586	.1575	.1562	.1546	.1529	.1511	.1490
7	.1399	.1418	.1435	.1450	.1462	.1472	.1480	.1486	.1489	.1490
8	.1066	.1099	.1130	.1160	.1188	.1215	.1240	.1263	.1284	.1304
9	.0723	.0757	.0791	.0825	.0858	.0891	.0923	.0954	.0985	.1014
10	.0441	.0469	.0498	.0528	.0558	.0588	.0618	.0649	.0679	.0710
11	.0245	.0265	.0285	.0307	.0330	.0353	.0377	.0401	.0426	.0452
12	.0124	.0137	.0150	.0164	.0179	.0194	.0210	.0227	.0245	.0264
13	.0058	.0065	.0073	.0081	.0089	.0098	.0108	.0119	.0130	.0142
14	.0025	.0029	.0033	.0037	.0041	.0046	.0052	.0058	.0064	.0071
15	.0010	.0012	.0014	.0016	.0018	.0020	.0023	.0026	.0029	.0033
16	.0004	.0005	.0005	.0006	.0007	.0008	.0010	.0011	.0013	.0014
17	.0001	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0006
18	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002
19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

1.6 POISSON DISTRIBUTION (Continued-II)

x	m									
	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	.0008	.0007	.0007	.0006	.0006	.0005	.0005	.0004	.0004	.0003
1	.0059	.0054	.0049	.0045	.0041	.0038	.0035	.0032	.0029	.0027
2	.0208	.0194	.0180	.0167	.0156	.0145	.0134	.0125	.0116	.0107
3	.0492	.0464	.0438	.0413	.0389	.0366	.0345	.0324	.0305	.0286
4	.0874	.0836	.0799	.0764	.0729	.0696	.0663	.0632	.0602	.0573
5	.1241	.1204	.1167	.1130	.1094	.1057	.1021	.0986	.0951	.0916
6	.1468	.1445	.1420	.1394	.1367	.1339	.1311	.1282	.1252	.1221
7	.1489	.1486	.1481	.1474	.1465	.1454	.1442	.1428	.1413	.1396
8	.1321	.1337	.1351	.1363	.1373	.1382	.1388	.1392	.1395	.1396
9	.1042	.1070	.1096	.1121	.1144	.1167	.1187	.1207	.1224	.1241
10	.0740	.0770	.0800	.0829	.0858	.0887	.0914	.0941	0967	.0993
11	.0478	.0504	.0531	.0558	.0585	.0613	.0640	.0667	.0695	.0722
12	.0283	.0303	.0323	.0344	.0366	.0388	.0411	.0434	.0457	.0481
13	.0154	.0168	.0181	.0196	.0211	.0227	.0243	.0260	.0278	.0296
14	.0078	.0086	.0095	.0104	.0113	.0123	.0134	.0145	.0157	.0169
15	.0037	.0041	.0046	.0051	.0057	.0062	.0069	.0075	.0083	.0090
16	.0016	.0019	.0021	.0024	.0026	.0030	.0033	.0037	.0041	.0045
17	.0007	.0008	.0009	.0010	.0012	.0013	.0015	.0017	.0019	.0021
18	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
19	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003	.0003	.0004
20	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002
21	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

x	m									
	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	.0003	.0003	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001
1	.0025	.0023	.0021	.0019	.0017	.0016	.0014	.0013	.0012	.0011
2	.0100	.0092	.0086	.0079	.0074	.0068	.0063	.0058	.0054	.0050
3	.0269	.0252	.0237	.0222	.0208	.0195	.0183	.0171	.0160	.0150
4	.0544	.0517	.0491	.0466	.0443	.0420	.0398	.0377	.0357	.0337
5	.0882	.0849	.0816	.0784	.0752	.0722	.0692	.0663	.0635	.0607
6	.1191	.1160	.1128	.1097	.1066	.1034	.1003	.0972	.0941	.0911
7	.1378	.1358	.1338	.1317	.1294	.1271	.1247	.1222	.1197	.1171
8	.1395	.1392	.1388	.1382	.1375	.1366	.1356	.1344	.1332	.1318
9	.1256	.1269	.1280	.1290	.1299	.1306	.1311	.1315	.1317	.1318
10	.1017	.1040	.1063	.1084	.1104	.1123	.1140	.1157	.1172	.1186
11	.0749	.0776	.0802	.0828	.0853	.0878	.0902	.0925	.0948	.0970
12	.0505	.0530	.0555	.0579	.0604	.0629	.0654	.0679	.0703	.0728
13	.0315	.0334	.0354	.0374	.0395	.0416	.0438	.0459	.0481	.0504
14	.0182	.0196	.0210	.0225	.0240	.0256	.0272	.0289	.0306	.0324
15	.0098	.0107	.0116	.0126	.0136	.0147	.0158	.0169	.0182	.0194
16	.0050	.0055	.0060	.0066	.0072	.0079	.0086	.0093	.0101	.0109
17	.0024	.0026	.0029	.0033	.0036	.0040	.0044	.0048	.0053	.0058
18	.0011	.0012	.0014	.0015	.0017	.0019	.0021	.0024	.0026	.0029
19	.0005	.0005	.0006	.0007	.0008	.0009	.0010	.0011	.0012	.0014
20	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0005	.0006
21	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0002	.0003
22	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001

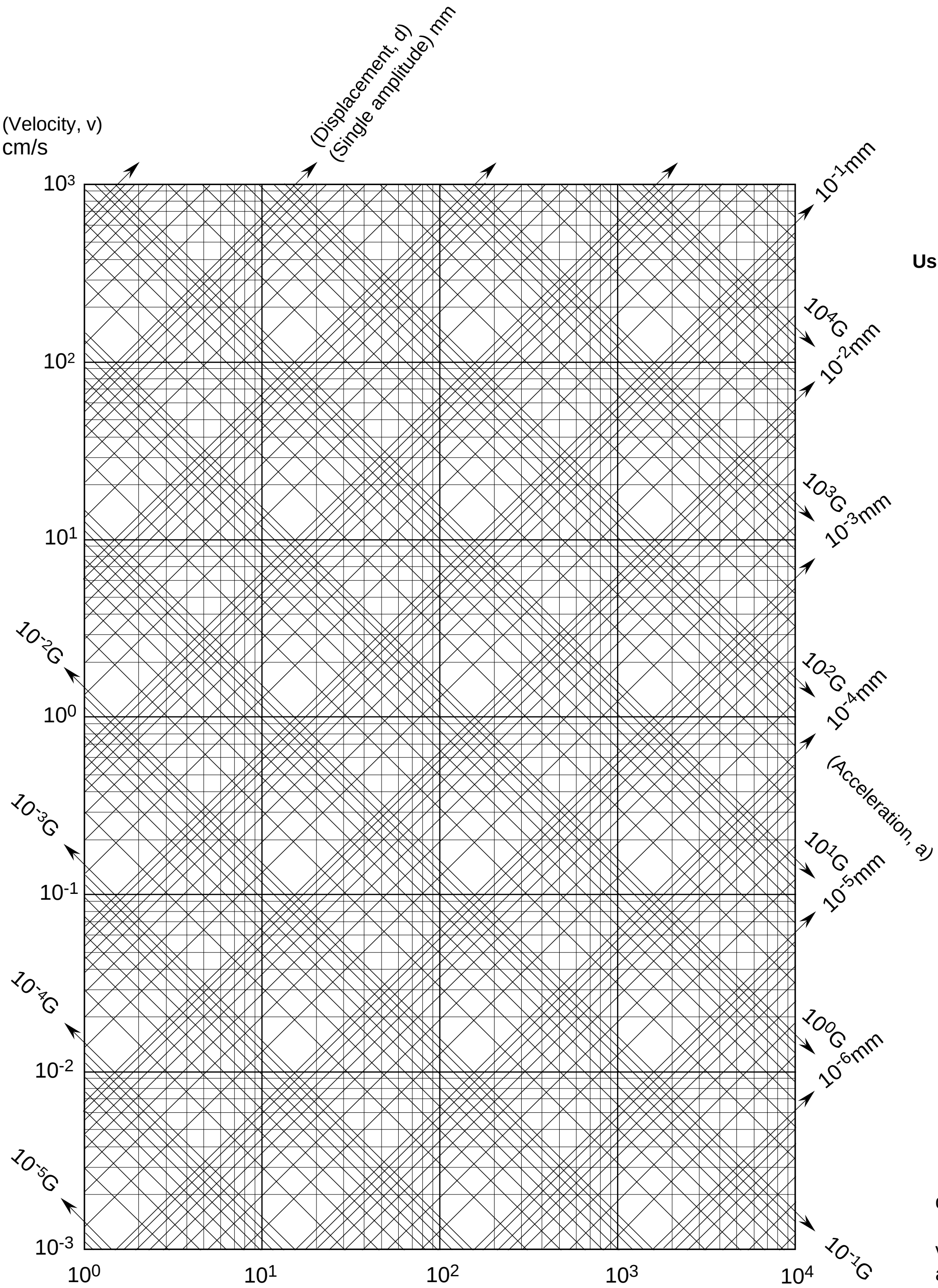
x	m									
	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
1	.0010	.0009	.0009	.0008	.0007	.0007	.0006	.0005	.0005	.0005
2	.0046	.0043	.0040	.0037	.0034	.0031	.0029	.0027	.0025	.0023
3	.0140	.0131	.0123	.0115	.0107	.0100	.0093	.0087	.0081	.0076
4	.0319	.0302	.0285	.0269	.0254	.0240	.0226	.0213	.0201	.0189
5	.0581	.0005	.0530	.0506	.0483	.0460	.0439	.0418	.0398	.0378
6	.0881	.0851	.0822	.0793	.0764	.0736	.0709	.0682	.0656	.0631
7	.1145	.1118	.1091	.1064	.1037	.1010	.0982	.0955	.0928	.0901
8	.1302	.1286	.1269	.1251	.1232	.1212	.1191	.1170	.1148	.1126
9	.1317	.1315	.1311	.1306	.1300	.1293	.1284	.1274	.1263	.1251

1.6 POISSON DISTRIBUTION (Continued-III)

x	m									
	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
10	.1198	.1210	.1219	.1228	.1235	.1241	.1245	.1249	.1250	.1251
11	.0991	.1012	.1031	.1049	.1067	.1083	.1098	.1112	.1125	.1137
12	.0752	.0776	.0799	.0822	.0844	.0866	.0888	.0908	.0928	.0948
13	.0526	.0549	.0572	.0594	.0617	.0640	.0662	.0685	.0707	.0729
14	.0342	.0361	.0380	.0399	.0419	.0439	.0459	.0479	.0500	.0521
15	.0208	.0221	.0235	.0250	.0265	.0281	.0297	.0313	.0330	.0347
16	.0118	.0127	.0137	.0147	.0157	.0168	.0180	.0192	.0204	.0217
17	.0063	.0069	.0075	.0081	.0088	.0095	.0103	.0111	.0119	.0128
18	.0032	.0035	.0039	.0042	.0046	.0051	.0055	.0060	.0065	.0071
19	.0015	.0017	.0019	.0021	.0023	.0026	.0028	.0031	.0034	.0037
20	.0007	.0008	.0009	.0010	.0011	.0012	0014	.0051	.0017	.0019
21	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
22	.0001	.0001	.0002	.0002	.0002	.0002	.0003	.0003	.0004	.0004
23	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

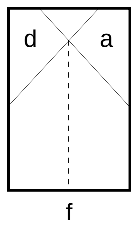
x	m									
	11	12	13	14	15	16	17	18	19	20
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2	.0010	.0004	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
3	.0037	.0018	.0008	.0004	.0002	.0001	.0000	.0000	.0000	.0000
4	.0102	.0053	.0027	.0013	.0006	.0003	.0001	.0001	.0000	.0000
5	.0224	.0127	.0070	.0037	.0019	.0010	.0005	.0002	.0001	.0001
6	.0411	.0255	.0152	.0087	.0048	.0026	.0014	.0007	.0004	.0002
7	.0646	.0437	.0281	.0174	.0104	.0060	.0034	.0018	.0010	.0005
8	.0888	.0655	.0457	.0304	.0194	.0120	.0072	.0042	.0024	.0013
9	.1085	.0874	.0661	.0473	.0324	.0213	.0135	.0083	.0050	.0029
10	.1194	.1048	.0859	.0663	.0486	.0341	.0230	.0150	.0095	.0058
11	.1194	.1144	.1015	.0884	.0663	.0496	.0355	.0245	.0164	.0106
12	.1094	.1144	.1099	.0984	.0829	.0661	.0504	.0368	.0259	.0176
13	.0926	.1056	.1099	.1060	.0956	.0814	.0658	.0509	.0378	.0271
14	.0728	.0905	.1021	.1060	.1024	.0930	.0800	.0655	.0514	.0387
15	.0534	.0724	.0885	.0989	.1024	.0992	.0906	.0786	.0650	.0516
16	.0367	.0543	.0719	.0866	.0960	.0992	.0963	.0884	.0772	.0646
17	.0237	.0383	.0550	.0713	.0847	.0934	.0963	.0936	.0863	.0760
18	.0145	.0256	.0397	.0554	.0706	.0830	.0909	.0936	.0911	.0844
19	.0084	.0161	.0272	.0409	.0557	.0699	.0814	.0887	.0911	.0888
20	.0046	.0097	.0177	.0286	.0418	.0559	.0692	.0798	.0866	.0888
21	.0024	.0055	.0109	.0191	.0299	.0426	.0560	.0684	.0783	.0846
22	.0012	.0030	.0065	.0121	.0204	.0310	.0433	.0560	.0676	.0769
23	.0006	.0016	.0037	.0074	.0133	.0216	.0320	.0438	.0559	.0669
24	.0003	.0008	.0020	.0043	.0083	.0144	.0226	.0328	.0442	.0557
25	.0001	.0004	.0010	.0024	.0050	.0092	.0154	.0237	.0336	.0446
26	.0000	.0002	.0005	.0013	.0029	.0057	.0101	.0164	.0246	.0343
27	.0000	.0001	.0002	.0007	.0016	.0034	.0063	.0109	.0173	.0254
28	.0000	.0000	.0001	.0003	.0009	.0019	.0038	.0070	.0117	.0181
29	.0000	.0000	.0001	.0002	.0004	.0011	.0023	.0044	.0077	.0125
30	.0000	.0000	.0000	.0001	.0002	.0006	.0013	.0026	.0049	.0083
31	.0000	.0000	.0000	.0000	.0001	.0003	.0007	.0015	.0030	.0054
32	.0000	.0000	.0000	.0000	.0001	.0001	.0004	.0009	.0018	.0034
33	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0010	.0020
34	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0012
35	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0007
36	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004
37	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

1.7 VIBRATION TABLES (amplitude, velocity, and acceleration vs. frequency)

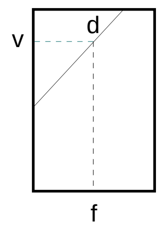


Using the Vibration Table

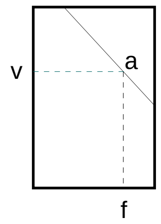
Relationship of d-a-f



Relationship of v-f-d



Relationship of v-f-a



d : displacement (mm) [single amplitude]
v : velocity (cm/sec)
a : acceleration (G)
f : frequency [Hz]

Equations

$a \fallingdotseq 0.004df^2$

$a \fallingdotseq 0.0066V \cdot f$

$d \fallingdotseq 250a/ f^2$

$d \fallingdotseq 1.6V/ f$

1.8 WATER VAPOR PRESSURE TABLES

SATURATED WATER VAPOR TABLE (by temperature)

Tempe-rature °C t	Saturation pressure Kg/cm^2 P_s	Tempe-rature °C t	Saturation pressure Kg/cm^2 P_s
0	0.006228	125	2.3666
5	0.008891	130	2.7544
10	0.012513	135	3.1923
15	0.017378	140	3.6848
20	0.023830	145	4.2369
25	0.032291	150	4.8535
30	0.043261	155	5.5401
35	0.057387	160	6.3021
40	0.075220	165	7.1454
45	0.097729	170	8.0759
50	0.12581	175	9.1000
55	0.16054	180	10.224
60	0.20316	185	11.455
65	0.25506	190	12.799
70	0.31780	195	14.263
75	0.39313	200	15.856
80	0.48297	210	19.456
85	0.58947	220	23.660
90	0.71493	230	28.534
95	0.86193	240	34.144
100	1.03323	250	40.564
105	1.2318	260	47.868
110	1.4609	270	56.137
115	1.7239	280	65.456
120	2.0245	290	75.915
		300	87.611

SATURATED WATER VAPOR TABLE (by pressure)

Pressure Kg/cm^2 P	Saturation pressure °C t_a	Pressure Kg/cm^2 P	Saturation pressure °C t_a
0.1	45.45	3.6	139.18
0.2	59.66	3.8	141.09
0.3	68.67	4.0	142.92
0.4	75.41	4.2	146.38
0.5	80.86	5.0	151.11
0.6	85.45	6	158.08
0.7	89.45	7	164.17
0.8	92.99	8	169.61
0.9	96.18	9	174.53
1.0	99.09	10	179.04
1.1	101.76	11	183.20
1.2	104.25	12	187.08
1.3	106.56	13	190.71
1.4	108.74	14	194.13
1.5	110.79	15	197.36
1.6	112.73	16	200.43
1.8	116.33	17	203.36
2.0	119.62	18	206.15
2.2	122.64	19	208.82
2.4	125.46	20	211.38
2.6	128.08	25	222.90
2.8	130.55	30	232.75
3.0	132.88	35	241.41
3.2	135.08	40	249.17
3.4	137.18	45	256.22
		50	262.70

(Source: The Japan society of machinery water vapor tables (new edition))
Note : 1 kg/cm^2 = 0.9678atm)

2. RELIABILITY THEORY

2.1 RELIABILITY CRITERIA

2.1.1 FAILURE RATE AND RELIABILITY FUNCTION

If we observe a sample of n devices at fixed time intervals h, we obtain a frequency distribution of the number of failures as shown in Fig. IX-1. In this analysis, r_i devices fail in the period t_i-t_{i-1}=h, with all devices failing before the time t_n.

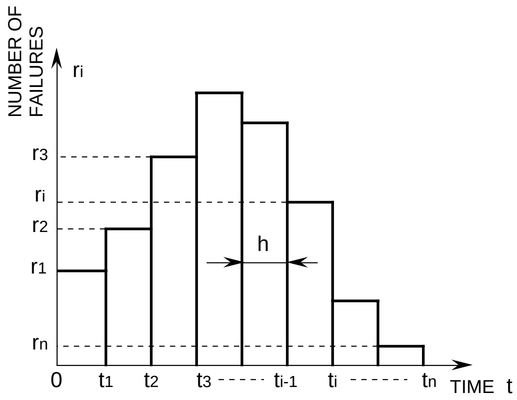


Fig. IX-1 Discrete Failure Distribution

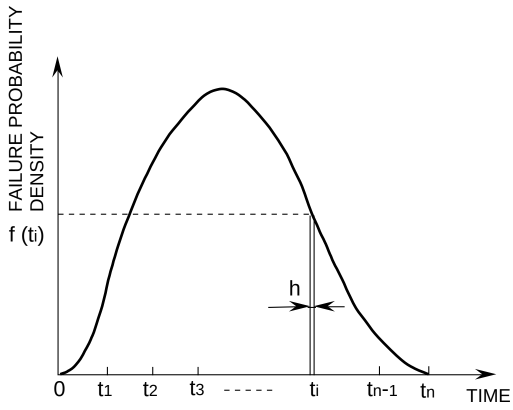


Fig. IX-2 Continuous Failure Distribution

The number of devices left after the i-th measurement period is given as $n_i = n - \sum_{i=1}^{i} r_i$. The average failure rate $\hat{\lambda}$ (t_{i-1}, t_i) in the period between t_{i-1} and t_i is given by the expression

$$\hat{\lambda}(t_{i-1}, t_i) = \frac{r_i}{n_{i-1}} \cdot \frac{1}{h} \qquad \text{(IX-1)}$$

If we make the time interval h increasingly small and use the failure rate density function f(t), the instantaneous failure rate λ (t_i, t_i+h) in the interval from t_i to t_i+h, shown in Fig. IX-2, is given by

$$\lambda(t_i, t_i+h) = \frac{f(t_i)\cdot h}{\int_{t_i}^{\infty} f(t)\,dt} \cdot \frac{1}{h} = \frac{f(t_i)}{\int_{t_i}^{\infty} f(t)\,dt} \qquad \text{(IX-2)}$$

Equation IX-2 is the generalization of the model in Fig. IX-2 with t_n as infinity.

The probability that a device will fail before the time t_i is known as the failure (or non-reliability) distribution function f(t_i). Also, the probability that a device will not fail before time t_i is the reliability function R(t_i). These functions are shown in Fig. IX-3.

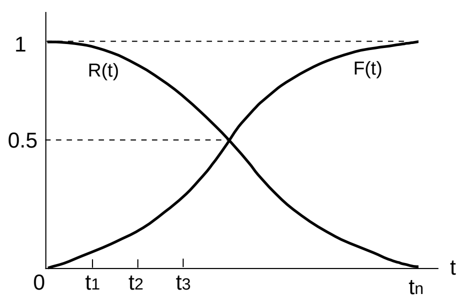


Fig. IX-3 Failure Distribution Function F(t) and Reliability Function R(t)

$$F(ti)=\int_{0}^{ti} f(t)dt \qquad \text{(IX-3)}$$

$$R(ti) = 1 - F(ti) = \int_{ti}^{\infty} f(t)dt \qquad \text{(IX-4)}$$

The probability P that a given semiconductor device will fail in the period between t and t+dt is the product of the probability R(t) that it will not fail before t and the instantaneous failure rate λ (t)dt in the period t to t+dt.

$$P= f(t)dt = R(t) \cdot \lambda (t)dt$$

$$\therefore \lambda (t)= \frac{f(t)}{R(t)} \qquad \text{(IX-5)}$$

For the failure rate λ (t), using the relationship of Eq. IX-4 we have

$$\lambda (t) = - \frac{1}{R(t)} \frac{d}{dt} R(t) = \frac{d}{dt} 1nR(t) \qquad \text{(IX-6)}$$

$$R(t) = \exp \left(-\int_{0}^{t} \lambda (t)dt\right) \qquad \text{(IX-7)}$$

2.1.2 DEFINITION OF RELIABILITY INDEX

Mean value (or expected value) μ and variance σ^2 (where σ is the standard deviation) are defined as characteristics of the distributions of Figs. IX-1 and IX-2 by the following expressions:

$$\left.\begin{aligned} \mu &= \int_{0}^{\infty} tf(t)dt \\ \sigma^2 &= \int_{0}^{\infty} (t-\mu)^2 f(t)dt = \int_{0}^{\infty} t^2 f(t)dt - \mu^2 \end{aligned}\right\} \text{(Continuous distribution)} \qquad \text{(IX-8)}$$

$$\left.\begin{aligned} \mu &= \sum_{i=0}^{\infty} ti\, f(ti) \\ \sigma^2 &= \sum_{i=0}^{\infty} (ti-\mu)^2\, f(ti) = \sum_{i=0}^{\infty} t_i^2\, f(ti) - \mu^2 \end{aligned}\right\} \text{(Discrete distribution)} \qquad \text{(IX-9)}$$

The expected life time (remaining life time) L(t) of a device which has been operated for time t is given by the following expressions:

$$\left.\begin{aligned} L(t) &= \frac{1}{R(g)} \left\{ \gamma - t + \int_{\gamma}^{\infty} R(t)dt \right\} && (t \le \gamma) \\ L(t) &= \frac{1}{R(t)} \int_{t}^{\infty} R(x)dx = \frac{1}{R(t)} \int_{0}^{\infty} R(t+y)dy && (\gamma \le t) \end{aligned}\right\} \qquad \text{(IX-10)}$$

Note that the device failure distribution function (Eq. IX-3) takes on the constant g in the range $0 \le t \le \gamma$, that is,

$$F(t) = 0 \text{ (where } 0 \le t \le \gamma)$$

and that after the passage of time γ it assumes a value (g is the order of position).

When equipment can be repaired by renewing a failed device, the mean value of the interval that operation is possible between occurrences of failures is know as the MTBF (Mean Time Between Failures).

If the operating time between subsequent failures of a device throughout its life until discarded is given by $t_1, t_2, ..., t_n$, the MTBF is given by

$$MTBF = \frac{t_1+t_2+ ... +t_n}{n} \tag{IX-11}$$

Since we are dealing with the measure of the entire operating life of the device, we will call ($t_1, t_2, ..., t_n$) a complete sample. MTBF known after the life of equipment is of no practical use. Therefore, the MTBF for a truncated portion of the life of the equipment up to the time T_0 is estimated using the following expression:

$$MTBF = \frac{t_1+t_2+ ... +t_r+(n-r)T_o}{n} \tag{IX-12}$$

In Eq. IX-12, r is the number of failures occurring until the time T_0.
We can also estimate the MTBF truncated after the number of failures = r.

$$MTBF = \frac{t_1+t_2+ ... +t_r+(n-r)t_r}{r} \tag{IX-13}$$

In Eqs. IX-12 and IX-13, the value n is determined by the type of failures for the device being considered (including such factors as the total number of semiconductor devices used and maximum number of failures before the equipment is disposed of).

In general, once a semiconductor device has failed, it cannot be repaired and used again. That is, it is a non-maintainable component. For this type of device, the mean time to the occurrence of a failure is know as the MTTF (Mean Time to Failure). As can be seen from Eq. IX-10, the remaining expected life L(t) is not equal to the MTTF minus the actual operating time. This is analogous to the fact that the remaining life of an adult is not necessarily equal to the expected life time of a new born child minus the adult's actual age.

In the period that failure rate λ (t) is time-independent and is constant, taking the value λ, we can use Eq. IX-7 to obtain

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t}\, dt = \frac{1}{\lambda} \tag{IX-14}$$

As a unit for the measure of failure rate,

$$1\times10^{-9} \text{ (failures/ (number of operating devices} \times \text{operating time))=1FIT} \tag{IX-15}$$

is used. For example, if we say that a given semiconductor device has a failure rate of 10 FIT, this means that one device fails for every 10^8 component hours. This, however, is not equivalent to saying the device life time is 10^8 hours. This is because the denominator of the defining expression Eq. IX-15 (component hours) does not refer to any particular device.

2.2 RELIABILITY OF COMPOSITE DEVICES

2.2.1 PARALLEL AND SERIES MODELS

Assume that we have n semiconductor devices used in series, and one device failure will result in the total group of devices failing. Such a system is known as a series system of redundancy of 0 (Fig. IX-4). If all the individual devices have failure mechanisms that are mutually independent, and the reliability function of the i-th device is given by the expression $R_i(t)$ (where i = 1, 2, ..., n), the reliability function R (t) of this series system is

$$R(t)=\prod_{i=1}^{n} R_i(t) \qquad \text{(IX-16)}$$

Equation VI-3 used for integrated circuit models in Section VI.2.2 can be modified to

$$\lambda p=C_1\pi_1 + C_2\pi_2$$

where $\pi_1= \pi Q \times \pi T \times \pi V \times \pi L$

$\pi_2 =\pi Q \times \pi E \times \pi L$

Then the reliability function Eq. IX-7 for integrated circuits can be expressed as follows using Eq. IX-16.

$$R(t) = e^{-\lambda pt}=e^{-C_1\pi_1 t} \cdot e^{-C_2\pi_2 t} =R_1(t) \cdot R_2(t)$$

where $R_1(t) = e^{-C_1\pi_1 t}$

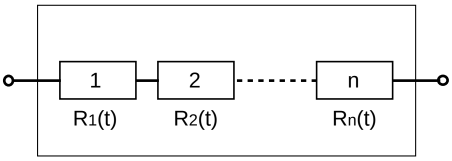
$R_2(t) = e^{-C_2\pi_2 t}$

Hence Eq. VI-3 is an equation derived by applying a series model of redundancy of 0 (Fig. IX-4) involving a failure factor (C_1) attributable to circuit complexity and a failure factor (C_2) caused by package complexity to integrated circuits.

There are other systems that n semiconductor devices are used in parallel, and as long as at least one of the devices is operating, the overall multiple device still functions. This is a parallel system with a redundancy of n –1 (Fig. IX-5). In this case as well, the failures of individual devices are taken to be mutually independent. If the failure distribution function for the i-th device is F_i (t), (where i = 1, 2, ..., n), the failure distribution probability function F (t) for the parallel system is

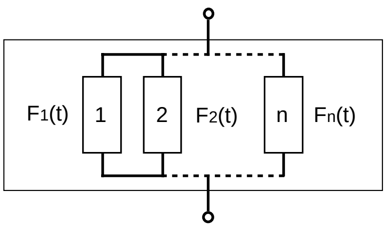
$$F(t)=\prod_{i=1}^{n} F_i (t) \qquad \text{(IX-17)}$$

$$R(t)=1-F (t) \qquad \text{(IX-18)}$$



$$R(t)=\prod_{i=1}^{n} R_i(t)$$

Fig. IX-4 Reliability Function for Series Model



R(t)=1–F(t)

Fig. IX-5 Reliability Function for Parallel Model

2.2.2 APPLICATION EXAMPLE

Let us consider the reliability function of the system in Fig. IX-6. This model consists of m units connected in series, which i-th unit has n_i devices connected in parallel. Moreover, we consider the system in Fig. IX-7, in which units of m series connected devices are connected in parallel up to n units. For the system in Fig. IX-6, let us assume that the reliability function for devices in the i-th unit is the same, which is $R_i(t)$. In Fig. IX-7, we assume that the reliability function for the i-th series connected devices R_{ij} (where j = 1, 2, ..., m) is the same and is $R_i(t)$. For the system in Fig. IX-6 we have

$$R(t)=\prod_{i=1}^{m}\{1-(R_i(t))^n\} \qquad \text{(IX-19)}$$

and for the system in Fig. IX-7 we have

$$R(t)=1-(1-\prod_{i=1}^{m}R_i(t))^n \qquad \text{(IX-20)}$$

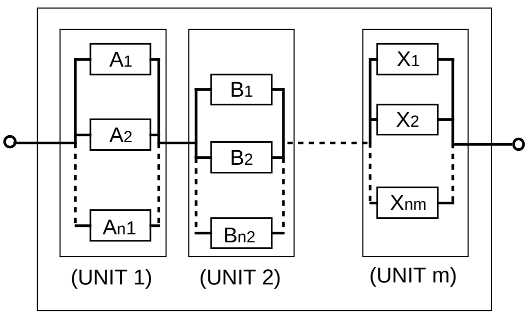


Fig. IX-6 Series-Parallel Composite Model (1)

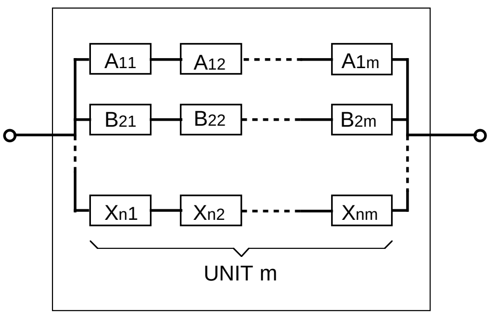


Fig. IX-7 Series-Parallel Composite Model (2)

2.2.3 STAND-BY REDUNDANCY SYSTEM

If we attach a selector switch to n devices in the parallel model shown in Fig. IX-5, we can select another device should one particular device experience a failure (Fig. IX-8).

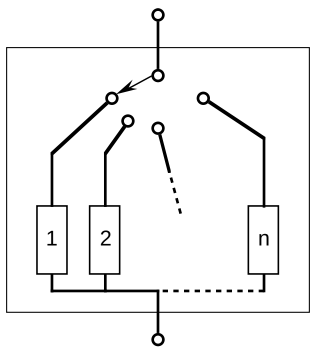


Fig. IX-8 Stand-by Redundancy Model

We will assume for simplicity that the switch does not fail and that the failure rates for the n devices are all equal to λ. The reliability function R (t) for this system is given by the Poisson partial sum.

$$R(t)=e^{-lt}\sum_{i=0}^{n-1}\frac{(\lambda t)^i}{i!} \qquad \text{(IX-21)}$$

An explanation of this equation will be given in Section 2.4.9, Poisson Distribution.

If we now assume that the failure rate of the switch is equal to λk for any tap, the overall reliability function becomes

$$R(t)=e^{-\{\lambda+(n-1)\lambda k\}t}\sum_{i=0}^{n-1}\frac{(\lambda t)^i}{i!} \qquad \text{(IX-22)}$$

2.3 FAILURE MODELS FOR ACCELERATED LIFE TESTING

2.3.1 REACTION THEORY MODEL

A particular characteristic of a device has the value X. Let us assume that the device will fail when the characteristic value changes to X_L. If the amount of change in the characteristic value is found to be accelerated by thermal stress, in many cases the Arrhenius chemical reaction kinetics model can be applied to this phenomenon.

In chemical reactions, if molecules reach the temperature above which they may react (the activation energy), a reaction occurs. The higher the temperature of the molecules, the higher becomes their energies, and so increasing the temperature quickens reactions. Arrhenius expressed the chemical reaction rate, K, experimentally as follows:

$$K = \Lambda e^{-\frac{\Delta E}{kT}} \qquad \text{(IX-23)}$$

where Λ : experimentally derived constant; k: Boltzmann's constant

ΔE : activation energy (kcal/mol); T: absolute temperature (K).

When considering the reliability of semiconductor devices, the activation energy is usually expressed in units of electron volts (eV), so that the Arrhenius relationship becomes

$$K = \Lambda e^{-\frac{\Delta E}{kT}} = \Lambda e^{11606\times\left(-\frac{B}{T}\right)} \qquad \text{(IX-24)}$$

where B : activation energy (eV).

1eV is equivalent to 23.05kcal/mol or 11,606K.

2.3.2 EYRING MODEL

The Eyring model is an extension of the Arrhenius model, and takes into consideration both mechanical stress and voltage stress as well as thermal stress. The reaction rate K using this model is given by the following expression:

$$K = A\left(\frac{kT}{h}\right)e^{-\frac{\Delta E}{kT}}\cdot e^{\left\{f(s)\cdot\left(C+\frac{D}{kT}\right)\right\}} \qquad \text{(IX-25)}$$

where A, C, and D: constants; ΔE : activation energy

k : Boltzmann's constant; T : absolute temperature (K)

h : Planck's constant; f(s) : stress function representing non-thermal stresses s

Here

$$f(s)=\ln s,\ C+\frac{D}{kT}=F$$

so that for small ranges of T, an approximation of Eq. IX-25 becomes

$$K = \Lambda T e^{-\frac{B}{T}} S^F \qquad \text{(IX-26)}$$

2.3.3 ACCELERATION FACTOR

Let us assume that an intermittent operation life test performed on a semiconductor device detected, as a result of the stress placed on the device, a leakage current which increases with time. As the device is subjected to more and more cycles of intermittent operation (n), as shown in Fig. IX-9, the leakage current increases. The level of degradation of the device can be expressed as a function of the leakage current i. If we take this current i as the device characteristic X discussed in Section 2.3.1, we can say that there is a failure criterion current $i_{R\,MAX}$ which corresponds to the device failure point X_L. That is,

$$f=f(i) \tag{IX-27}$$

Since the reaction rate K in the accelerated life test is basically defined as the rate of degradation of the device, we have

$$K=\frac{df(i)}{dt} \tag{IX-28}$$

$$\therefore f(i)=Kt \tag{IX-29}$$

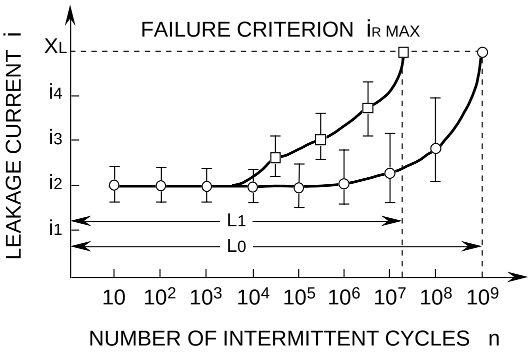


Fig. IX-9 Data Example for Intermittent Operation Life Test

The pattern of degradation f caused by the intermittent test may vary depending upon the current flowing in the device. In Fig. IX-9 the curve marked with circles represents such test performed at the rated current of the device, while that marked with squares shows the results of operating the device intermittently at 1.5 times the rated current. Since in either case the device fails when $i = i_{R\,MAX}$, if we use the subscripts 0 and 1 to represent test conditions for these two cases, we have

$$L_0=\frac{f(i_{R\,MAX})}{K_0}, \qquad L_1=\frac{f(i_{R\,MAX})}{K_1} \tag{IX-30}$$

In this case, the current acceleration factor α_1 is

$$\alpha_1=\frac{L_0}{L_1}=\frac{K_1}{K_0} \quad \text{(where the subscript 0 represents operation at the rated current)} \tag{IX-31}$$

For the purposes of the explanation we have used an actual example, as shown in Fig. IX-9. We can, however, make a generalization about the state function f(X) and the characteristic value X that defines the state. If we use the relationship in Eq. IX-29 in the Arrhenius equation of Eq. IX-24, we have

$$K=\Lambda e^{-\frac{\Delta E}{kT}}=\frac{f(X)}{X} \tag{IX-32}$$

If we assume that the device reaches the end of its life when the characteristic value $X = X_L$, as discussed in Section 2.3.1, Eq. IX-32 will be

$$K = \frac{f(X_L)}{L} \qquad \text{(IX-32')}$$

Then we have the relationship between temperature and life as

$$\ln L = \ln f(X_L) - \ln \Lambda + \frac{\Delta E}{kT} \qquad \text{(IX-33)}$$

Life tests using temperature as a dominant factor are verified by a logarithmic normal distribution (Eq. IX-79). This is based on Eq. IX-33. If we let T_0 and L_0 be the reference conditions (such as the standard operating conditions) for temperature and life, and T_1 and L_1 be the corresponding temperature and life for accelerated conditions, the temperature acceleration factor α_T is referring to Eq. IX-32,

$$\alpha T = \frac{L0}{L1} = e^{\frac{\Delta E}{k}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)} \qquad \text{(IX-34)}$$

As can be seen by Eq. IX-34, acceleration caused by heat varies depending on the activation energy ΔE. The relationship between activation energy and the acceleration factor is shown in Fig. IX-10.

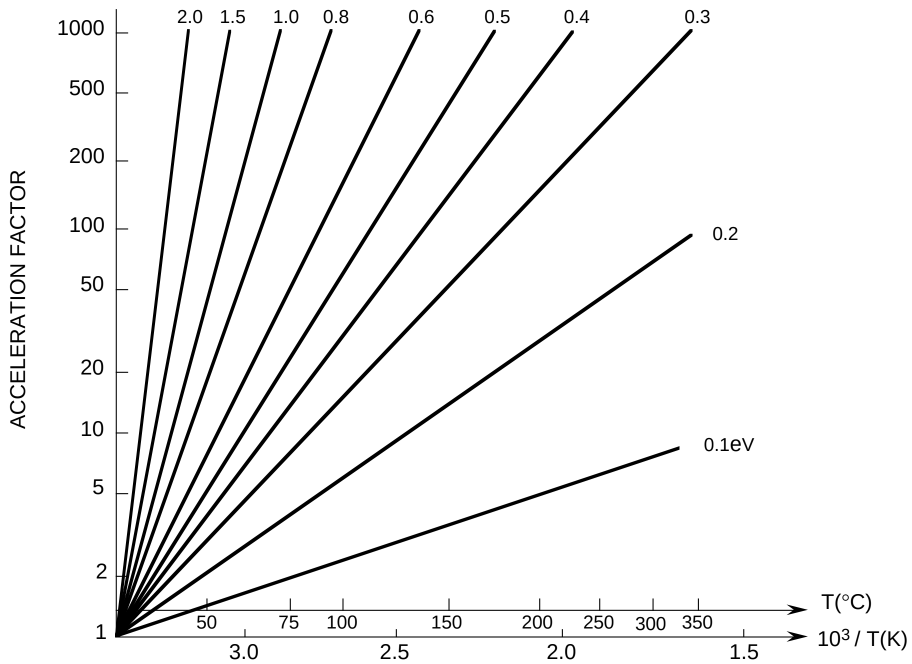


Fig. IX-10 Activation Energy Versus Acceleration Factor

2.4 PROBABILITY MODELS USED IN RELIABILITY ANALYSIS

2.4.1 BERNOULLI TRIAL

From a given population of semiconductor devices a single device is sampled and tested. The possible test results are limited to either ① "failure or defect"or ② "no failure or acceptance," with no possibility of such results as "pending decision" or "exception acceptance" allowed. This cycle of sampling, testing, and rejection/acceptance is repeated n times. In a single such test the probability of ① "failure" is p and the probability of ② "no failure" is q (p + q = 1). The values of p and q will be uniform for all test results. Each test result is independent from one another. This discrete model is termed the Bernoulli trial or sampling. For ease of understanding we have chosen "failure" and "no failure" for results ① and ②, respectively. The fundamental condition of the Bernoulli trial is that results are only two types and they are definitely identified.

2.4.2 BINOMIAL DISTRIBUTION $f_{Bin}(x, n, p)$

In the Bernoulli trial, assume that x of n tests result in ① and n – x tests result in ② . Such a phenomenon occurs with some probability. This probability is described by the binomial probability distribution $f_{Bin}(x, n, p)$.

$$\left.\begin{aligned} & f_{Bin}(x,n,p) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!}\, p^x q^{n-x} \\ & \mu = np, \quad \sigma^2 = npq \end{aligned}\right\} \qquad \text{(IX-35)}$$

A sample lot comprising n devices randomly selected from a large population whose average fraction defective is p, contains x defective devices with some probability. Such a probability is a typical example of the binomial probability distribution.

2.4.3 NEGATIVE BINOMIAL DISTRIBUTION $f_{neg-bin}(x, n, p)$ and MULTINOMIAL DISTRIBUTION $f_{multi-bin}(x_1, x_2, ..., x_m, n, p_1, p_2, ..., p_m)$

Let us consider the number of tests n required before we encounter ① "failure" x times in the Bernoulli trial. By the (n – 1) th test there have been x – 1 times of ① and (n – 1) – (x – 1) = n – x times of ② , and the x-th failure occurs on the n-th test. The probability of this, $f_{neg-bin}(x, n, p)$ is

$$f_{neg-bin}(x,n,p) = \binom{n-1}{x-1} p^{x-1} \cdot q^{(n-1)-(x-1)} \cdot p = \binom{n-1}{n-x} p^x q^{n-x} \qquad \text{(IX-36)}$$

Using the characteristics of binomial coefficients, we have

$$\binom{-x}{x-1} = (-1)^{n-x} \binom{n-1}{n-x}$$

Hence from Eq. IX-36,

$$\begin{aligned} \sum_{n=x}^{\infty} f_{neg-bin}(x,n,p) &= \sum_{n=x}^{\infty} \binom{n-1}{n-x} p^x q^{n-x} = p^x \sum_{n=x}^{\infty} (-1)^{n-x} \binom{-x}{n-x} q^{n-x} \\ &= p^x \sum_{r=0}^{\infty} \binom{-x}{r} (-q)^r = p^x (1-q)^{-x} = 1 \end{aligned} \qquad \text{(IX-37)}$$

Since $f_{neg-bin}(x, n, p) \geq 0$, we have, from Eq. IX-37

$$f_{neg-bin}(x,n,p) = \binom{n-1}{n-x} p^x q^{n-x} p = \binom{-x}{n-x} p^x q^{n-x}, \quad \mu = \frac{xq}{p}, \quad \sigma^2 = \frac{xq}{p^2} \tag{IX-38}$$

There is a case that test results are not limited to two possible results of acceptance and rejection but fall into m classes ($E_1, E_2, ..., E_m$). Let us examine the probability $f_{multi-bin}(x_1, x_2, ..., x_m, n, p_1, p_2, ..., p_m)$ with which tested devices fall into these classes. After n times of tests, any one of the results is $E_1, E_2, ...,$ or E_m. The result E_i occurs with the probability p_i. The result E_i is observed x_i times ($\sum_{i=l} x_i = n, n \geq x_i \geq 0$) during n tests. Hence, results fall into m classes ($E_1, E_2, ..., E_m$) numbering $x_1, x_2, ..., x_m$ with the probability $f_{multi-bin}(x_1, x_2, ..., x_m, n, p_1, p_2, ..., p_m)$. This is known as the multinomial distribution.

$$f_{multi-bin}(x_1, x_2, ... x_m, n, p_1, p_2 ... p_m) = \frac{n!}{x_1!\, x_2! ... x_m!} p_1^{x_1} p_2^{x_2} ... p_m^{x_m} \tag{IX-39}$$

The multinomial distribution is an extended binomial distribution with m variables.

2.4.4 GEOMETRIC DISTRIBUTION $f_{Geo}(n, p)$

In the Bernoulli trial, the first ① "failure" is encountered on the n-th test. The probability of this phenomenon is expressed by the geometric distribution $f_{Geo}(n, p)$

$$f_{Geo}(n, p) = q^{n-1} p \ (n = 1,2,...), \quad \mu = \frac{q}{p}, \quad \sigma^2 = \frac{q}{p^2} \tag{IX-40}$$

The geometric distribution is a case of Eq. IX-38 for the negative binomial distribution where x = 1. The mean value of the geometric distribution μ is the expected value for the number of tests in Bernoulli trial before the first occurrence of ① . The failure distribution function of the geometric distribution is

$$F_{Geo}(N, p) = \sum_{n=1}^{N} f_{Geo}(n, p) = p + qp + q^2 p + ... + q^{N-1} p = 1 - q^N$$

which indicates that even if the acceptance rate q on any particular test is high ($q \simeq 1$), a failure will eventually occur with a larger number of tests N ($\lim_{N\to\infty} F_{Geo}(N, p) = 1$).

2.4.5 HYPERGEOMETRIC DISTRIBUTION $f_{H-geo}(N, R, n, x)$

In the mass production of semiconductor devices a widely used technique is the sampling of a small number of devices n from a large population of N devices and making decision on the total population based on observation of the sample alone. An overall population of N devices has R defective devices (R is not known unless 100% inspection is performed). By inspecting n randomly sampled devices, x defective devices are detected with the probability $f_{H-geo}(N, R, n, x)$. If min (R, n) represents the smaller of R and n, we have

$$f_{H-geo}(N,R,n,x) = \frac{\binom{R}{x}\binom{N-R}{n-x}}{\binom{N}{n}} \ (0 \leq x \leq \min(R,n)), \quad \mu = n\frac{R}{N} = np, \quad \sigma^2 = \frac{N-n}{N-1} npq \tag{IX-41}$$

This is referred to as the probability function of hypergeometric distribution. Different ways are available for sampling n devices from N devices. If the sampled device is returned each time to choose a device from N constantly, or using replacement operations in other words, we will obtain a binomial distribution. If the sampled device is not returned (non-replacement), we will have a hypergeometric distribution. If the original population is large, however, the fraction defective obtained through a sampling inspection can be approximated by the binomial distribution. By expanding Eq. IX-41 and reordering the product terms, we have

$$f_{H-geo}(N,R,n,x) = \binom{n}{x} \prod_{j=0}^{x-1} \frac{R-i}{N-i} \cdot \prod_{j=0}^{n-x-1} \left(1 - \frac{R-x}{N-x-j}\right) \qquad \text{(IX-42)}$$

If N and R increase to infinity while maintaining $\frac{R}{N}$ = p constant, for finite n (and therefore finite x) we have

$$\lim_{N,R\to\infty} \prod_{i=0}^{x-1} \frac{R-i}{N-i} = \left(\frac{R}{N}\right)^x = p^x, \quad \lim_{N,R\to\infty} \prod_{j=0}^{n-x-1} \left(1 - \frac{R-x}{N-x-j}\right) = \left(1 - \frac{R}{N}\right)^{n-x} = (1-p)^{n-x}$$

Using this relation and letting N and R approach infinity to the limit, from Eq. IX-42 we obtain

$$f_{H-geo}(N,R,n,x) \to \binom{n}{x} p^x (1-p)^{n-x} = f_{Bin}(x,n,p) \qquad \text{(IX-43)}$$

An actual sampling plan sets a rejection criterion number c. If the number of defective devices x detected in a sample of n devices does not exceed c, the entire lot is considered to have passed inspection. When n devices are sampled from a population with fraction defective p (= $\frac{R}{N}$) the number of defective devices x will not exceed c with some probability. This probability ψ with which the lot is judged to be accepted (the lot acceptance rate) is given by Eq. IX-44.

$$\psi = \sum_{x=0}^{c} f_{H-geo}(N,Np,n,x) = \sum_{x=0}^{c} \frac{\binom{Np}{x}\binom{Nq}{n-x}}{\binom{N}{Np}} \qquad \text{(IX-44)}$$

The lot acceptance rate ψ varies depending on how the values of n and c are chosen for populations of the same quality level (i.e., p being the same value), as is clearly indicated by Eq. IX-44. How the lot acceptance rate changes is illustrated by the operation characteristic curve (OC curve) in Fig. IX-11. The fraction defective p (= $\frac{R}{N}$) with which the population contains defective devices is plotted on the horizontal axis. The probability ψ with which the lot is judged to be accepted through sampling inspection is plotted on the vertical axis.

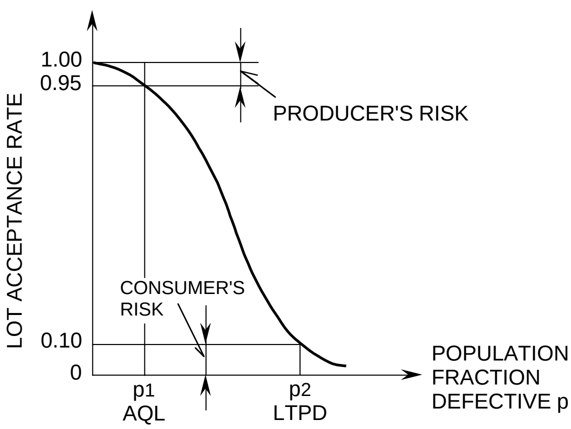


Fig. IX-11 OC Curve

In some sampling plans, the fraction defective p_1 is set to control the lot acceptance rate to, for example, 95%. This plan is known as the AQL (Acceptable Quality Level) plan. In this case, 5% of lots whose quality level could be accepted are rejected by sampling inspection. The producer gives up shipping the 5%. The risk of this rejection implies the producer's loss, so the risk is referred to as producer's risk.

In some other inspection plans, lots whose fraction defective is p_2 are accepted with a probability of, for example, 10%. Such an inspection plan is known as the LTPD (Lot Tolerance Percent Defective) plan. This implies that the consumer takes a risk of purchasing a lot whose fraction defective is p_2 with a probability of 10%. This is known as consumer's risk.

The AQL plan measures the lot whose fraction defective is p_1 as having the lowest acceptable quality level. In contrast, the LTPD plan verifies that the fraction defective is no more than p_2 with 90% probability.

2.4.6 EXPONENTIAL DISTRIBUTION f_{exp} (t)

The failure rate is constant with time in the random failure period. Therefore, from Eq. IX-7, we have

$$R(t)=e^{-\lambda t} \quad \text{(IX-45)}$$

$$\left.\begin{array}{l} f_{exp}(t)=\lambda e^{-\lambda t} \\ \mu=\frac{1}{\lambda}=\text{MTTF (or MTBF)}, \quad \sigma^2=\frac{1}{\lambda^2} \end{array}\right\} \quad \text{(IX-46)}$$

Observation of a model with a constant failure rate may be either continuous or discrete at fixed intervals. By considering the probability of detecting the first failure at time t, the relation of exponential and geometric distributions can be identified.

Let us assume that floating dust causes an average of r mask defects on the surfaces of silicon wafers of a given lot. The area of the wafer surface S is divided into many portions. From one end of the wafer, each portion is inspected with a microscope. Since the location of the dust particles on the wafer surface is unpredictable, we can assume that mask defects occur completely at random. Therefore, the failure rate λ is uniform over the wafer's entire surface, so $\lambda=\frac{r}{S}$, The probability p of a mask defect existing in a divided portion is

$$p = C\lambda \quad \text{(IX-47)}$$

where C is the area of the portion.

If the first detection of a mask defect is in the x-th portion, that is, the first detection occurs when the inspected area reaches t (t = Cx), the probability that this phenomenon will occur is expressed by the geometric distribution $pq^{x-1} = p(1-p)^{x-1}$ on the average in the portion C that includes t. A mask defect can be detected at the same level of expectation at the inspection area t and at the number of inspection cycles x (Fig. IX-12). Therefore

$$\text{Mask defect expectation} = \lambda t = xp \quad \text{(IX-48)}$$

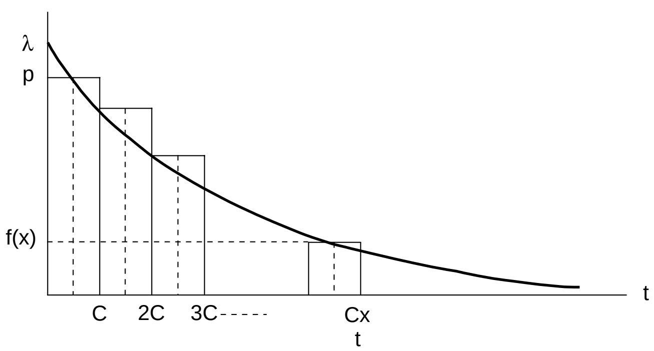


Fig. IX-12 Relation between Geometric and Exponential Distributions

Incidentally, continuous observation is in this case equivalent to reduction of the area C to the limit. The average probability of finding a mask defect within the area C approaches the probability that a mask defect exists on a point on the surface of the wafer. Noting the relations expressed by Eqs. IX-47 and IX-48, we have

$$\lim_{C\to 0}\frac{1}{C}\,pq^{\,x-1} = \lim_{C\to 0}\frac{p}{C}(1-p)^{x-1} = \lambda \lim_{p\to 0}\left\{(1-p)^{\frac{1}{p}}\right\}^{\lambda t}\cdot\frac{1}{1-p} = \lambda\, e^{-\lambda t} \qquad \text{(IX-49)}$$

This implies that the exponential distribution is equivalent to the limit of the geometric distribution.

2.4.7 PASCAL DISTRIBUTION $f_{Pas}(x, y, p)$

In a Bernoulli trial comprising n = x + y tests, there may be a case that the last n-th test result is ① "failure" after x times of ① "failure" and y times of ② "no failure." The probability that this phenomenon occurs is expressed by the Pascal probability distribution $f_{Pas}(x, y, p)$.

$$\left.\begin{aligned} f_{Pas}(x,y,p) &= \binom{x+y-1}{y} p^{x} q^{y} \\ \mu = \frac{qy}{p}, &\quad \sigma^2 = \frac{qy}{p^2} \end{aligned}\right\} \qquad \text{(IX-50)}$$

If x = 1 in a specific case, the Pascal distribution is the same as the geometric distribution.

$$f_{Pas}(1,y,p) = pq^{\,y} = f_{Geo}(y+1,p) \qquad \text{(IX-51)}$$

Consider that products are randomly sampled from a flow process in a production line for the purpose of intermediate inspection. The probability of defective items produced in the manufacturing process is p. For a sampling inspection of n = x + r items, the probability that the r-th failed product is detected is expressed by the Pascal distribution.

We can find a relation between the Pascal distribution and the binomial distribution if we interpret the Pascal distribution as a Bernoulli trial comprising n – 1 tests with phenomenon ① invariably occurring at the last n-th time after x – 1 times of phenomenon ① .

$$f_{Pas}(x,y,p) = p\, f_{Bin}(x-1, n-1, p), \quad \text{where } n = x+y \qquad \text{(IX-52)}$$

2.4.8 GAMMA DISTRIBUTION $f_{\Gamma}(t, \alpha, \beta)$

We can derive the relation below from Eq. IX-50 representing the Pascal distribution.

$$f_{Pas}(x,y,p) = \frac{p(n-1)}{x-1} f_{Pas}(x-1, y, p), \quad \text{where } n = x+y \qquad \text{(IX-53)}$$

From Eqs. IX-51 and IX-49, we have

$$f_{Pas}(1,y,p) = f_{Geo}(y+1, p) \;\to\; f_{exp}(\lambda t) = \lambda\, e^{-\lambda t} \qquad \text{(IX-54)}$$

In Eq. IX-54, the arrow → denotes the operation $\lim_{C\to 0}$ used to reduce the divided portion C for discrete observation, which is the relation shown by Eq. IX-49. Noting the relation expressed by Eqs. IX-53 and IX-54, we can obtain Pascal probability distribution functions respectively for x = 1, 2, 3, ...

$$x = 1: f_{Pas}(1,y,p) \;\to\; \lambda e^{-\lambda t} \qquad \text{(IX-55)}$$

$$x = 2: f_{Pas}(2,y,p) = \frac{p(n-1)}{1} f_{Pas}(1, y, p) = \frac{\lambda t}{1} f_{Pas}(1, y, p) \to \frac{\lambda t}{1}\lambda\, e^{-\lambda t} \qquad \text{(IX-56)}$$

In Eq. IX-56, we used the relation represented by Eq. IX-48. Similarly we have

$$x=3 : f_{Pas}(3,y,p)= \frac{\lambda t}{1}\frac{\lambda t}{2}\, f_{Pas}(1, y, p) \rightarrow \frac{(\lambda t)^2}{2\,!}\lambda e^{-\lambda t}$$

Its generalized equation is

$$f_{Pas}(x,y,p) \rightarrow \lambda\, \frac{(\lambda t)^{x-1}}{(x-1)\,!}\, e^{-\lambda t} \equiv f_{\Gamma}(t,x,\lambda) \qquad \text{(IX-57)}$$

The function $f_{\Gamma}(t, x, \lambda)$ derived by Eq. IX-57 from $f_{Pas}(x, y, p)$ is known as the Gamma probability density function. By calculating the normalized constant of $\int_0^\infty dt$ for Eq.IX -57, we obtain the generalized expression of the Gamma distribution.

$$f_{\Gamma}(t, \alpha, \beta) = \frac{\alpha}{\Gamma(\beta)}\,(\alpha t)^{\beta-1} e^{-\alpha t} \qquad \text{(IX-58)}$$

In Eq. IX-58 expressing the Gamma distribution, β is the shape parameter and α, the scale parameter. Figure IX-13 shows graphs of Eq. IX-57 where $\lambda = 1$.

$$f_{\Gamma}(t,x,1)= \frac{1}{(x-1)\,!}\, t^{x-1} e^{-t} \qquad \text{(IX-59)}$$

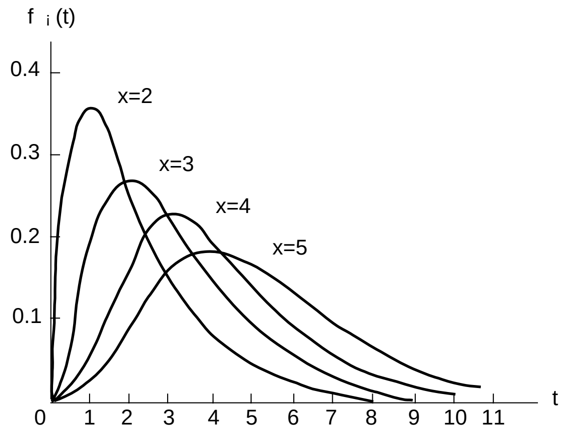


Fig. IX-13 Gamma Probability Density Functions $f_{\Gamma}(t,x,1)$

As can be seen from the derivation of the Gamma distribution given by Eqs. IX-55 through IX-57, the Pascal and Gamma distributions are the discrete and continuous probability distributions for the same probability model.

In particular, for the case of x = 1, the Gamma distribution (IX-57) reduces to the exponential distribution (IX-46). The Pascal distribution, if x = 1, becomes the geometric distribution (IX-51). This is consistent with the fact that the geometric and exponential distributions are discrete and continuous distributions, respectively (IX-49). The Gamma distribution is a failure probability density function if failure occurrence is considered to follow the Poisson process. We deal with this issue in Section 2.4.9, Poisson Distribution.

2.4.9 POISSON DISTRIBUTION $f_{Pois}(x)$

Cosmic rays collide with semiconductor devices used in an artificial satellite completely at random. We cannot expect that there will be no collision for some time since one has just occurred. Similarly, we cannot say that a cosmic ray will soon collide with a semiconductor device because there have been no collisions for a moment.

This is an example of cases in which we can consider that a rare phenomenon will occur with some expectation if the period of observation is sufficiently long or the population to be observed is sufficiently large. Here we assume that the phenomenon does not occur twice or more at the same instant, and further that the probability of the phenomenon occurring is constant. In other words, the MTTF of the phenomenon = θ (or the instantaneous failure rate = λ) is constant. Such a probability process is known as the Poisson process.

If a phenomenon occurs in accordance with the Poisson process with the expectation x, the probability of the phenomenon occurring n times within the time interval $0 < T \leq t_1$ is described by the Poisson distribution. Detailed discussions of the Poisson distribution are given below.

Let us determine the probability that the phenomenon occurs in a short moment dt in the time $0 < T \leq t_1$. Using reliability functions applicable to times $0 < T \leq t_1$ and $0 < T \leq t_1 - t$ before and after dt, we obtain

$$R(t) \cdot \frac{dt}{\theta} \cdot R(t_1 - t) \qquad \text{(IX-61)}$$

Hence the probability $P_1\left(\frac{t_1}{\theta}\right)$ of the phenomenon occurring once in the time $0 < T \leq t_1$ is

$$P_1\left(\frac{t_1}{\theta}\right) = \int_0^{t_1} R(t)\,\frac{dt}{\theta}\,R(t_1 - t) = \int_0^{t_1} e^{-\frac{t}{\theta}} \cdot \frac{dt}{\theta} \cdot e^{-\frac{t_1 - t}{\theta}} = \frac{1}{\theta}\,e^{-\frac{t_1}{\theta}} \cdot t_1 \qquad \text{(IX-62)}$$

The probability $P_n\left(\frac{t_1}{\theta}\right)$ that the phenomenon occurs n times within $0 < T \leq t_1$ is obtained through repeated calculations of Eq.

$$P_n\left(\frac{t_1}{\theta}\right) = \int_0^{t_1} R(t)\,\frac{dt}{\theta}\,P_{n-1}\left(\frac{t_1}{\theta}\right) = \frac{1}{n\,!}\left(\frac{t_1}{\theta}\right)^n e^{-\frac{t_1}{\theta}} \qquad \text{(IX-63)}$$

Eventually, when the expectation $x = \frac{t_1}{\theta}$, the Poisson probability density function $f_{Pois}(x)$ is

$$\left.\begin{aligned} f_{Pois}(x) &= \frac{1}{n\,!}\,x^n e^{-x} \\ \mu = x, &\quad \sigma^2 = x^2 \end{aligned}\right\} \qquad \text{(IX-64)}$$

A three-dimensional representation of Eq. IX-64 is shown in Fig. IX-14 within the range of $1 \leq n \leq 6$ and $0 \leq x \leq 6$. The values of f(x) for representative values in the range of

$$0 \leq n \leq 39, \quad \text{and} \quad 0.1 \leq x \leq 20$$

are shown in the attached table

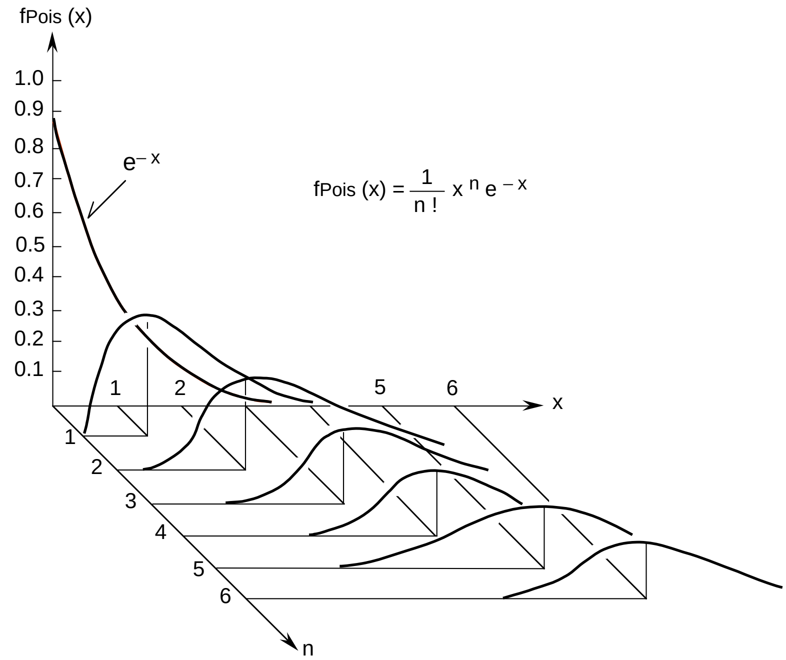


Fig. IX-14 Three-Dimensional Representation of Poisson Distributions

Let us determine the probability $F(t_1, k)$ that failures occur at least k times or more within the time $0 < T \leq t_1$ in a Poisson process. This probability is expressed as follows using Eq. IX-63 for the Poisson distribution.

$$F(t_1, k)=1-\sum_{n=0}^{k-1} Pn\left(\frac{t_1}{\theta}\right)=1-\sum_{n=0}^{k-1}\frac{1}{n!}\left(\frac{t_1}{\theta}\right)^n e^{-\frac{t_1}{\theta}} \tag{IX-65}$$

In Section 2.2.3, the stand-by redundancy system in Fig. IX-8 will not fail unless the n switches all fail. Therefore, based on Eq. IX-65, the reliability function R(t) of this stand-by redundancy system is

$$R(t)=1-F(t, k)$$

This equation leads to Eq. IX-21.

If damage causing failure occurs randomly in accordance with the Poisson process, the failure distribution function of the device is Eq. IX-65 assuming that the device breaks from k times or more of damage in the operating time $0 < T \leq t_1$. In this case, the failure probability density function becomes Eq. IX-57 for the Gamma distribution. This is explained as follows.

Using a fixed value, in Eq. IX-65, for the number of damages k received before failure, consider the failure distribution function $F(t_1, k)$ as a function of time t_1. Based on Eq. IX-3, the failure probability density function $f(t_1)$ is

$$f(t_1)=\frac{\alpha}{\alpha t_1}F(t_1, k)=\frac{1}{(k-1)!}\left(\frac{t_1}{\theta}\right)^{k-1} e^{-\frac{t_1}{\theta}}=f\Gamma\left(t_1, x, \frac{1}{\theta}\right) \tag{IX-66}$$

Using a fixed value, in Eq. IX-65, for the number of damages k received before failure, consider the failure distribution function $F(t_1, k)$ as a function of time t_1. Based on Eq. IX-3, the failure probability density function $f(t_1)$ is

If, on the other hand, only a single damage (n = 1) is fatal to the device, the failure probability density function becomes Eq. IX-46 of the exponential distribution.

The Poisson distribution approximates the binomial probability distribution if the population is large and the phenomenon occurs with a low probability.

The binomial probability distribution should deal with Bernoulli samples, for which the probability p = constant is a premise. In contrast, the Poisson distribution handles phenomenons that the expectation x = Np = constant. It is important to note that N no longer denotes the number of tests of Bernoulli trial due to the approximation of the binomial probability distribution to the Poisson distribution. This process is explained in detail as follows.

$$f_{Bin}(n,N,P)=\frac{N!}{n!(N-n)!}p^n q^{N-n}=\frac{\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)\ldots\left(1-\frac{n-1}{N}\right)}{n!}(Np)^n q^{N-n} \tag{IX-67}$$

and,

$$\log q^{N-n}=(N-n)\log(1-p)=-(N-n)\sum_{k=1}^{\infty}\frac{p^k}{k}=-\left(1-\frac{n}{N}\right)\left(x+\frac{1}{2}\frac{x^2}{N}+\frac{1}{3}\frac{x^3}{N^2}+\ldots\right) \tag{IX-68}$$

$$\therefore \lim_{N\to\infty}\log q^{N-n}=-x,\qquad \lim_{N\to\infty}q^{N-n}=e^{-x} \tag{IX-69}$$

N in these equations is not the number of tests of Bernoulli trial, but serves to indicate that the population size is expanded. Through these steps we obtain

$$\lim_{N\to\infty}f_{Bin}(n,N,P)=\frac{1}{n!}x^N e^{-x} \tag{IX-70}$$

Thus understanding the Poisson distribution in different ways we can clarify its relations with other probability distributions. This will be illustrated later in "Relations of Probability Distributions."

2..4.10 NORMAL DISTRIBUTION f_{Norm} (x)

The general character of a population comprising numerous, uniform, and random independent phenomena is expressed by the normal distribution f_{Norm} (x). f_{Norm} A typical example of this is the movement of molecules of a classical, ideal gas. f_{Norm} (We will not discuss the correctness of this in a strict mathematical sense.)

Expected characteristics of the function fNorm (x) are these: the mean value μ is also the maximum value; the value smoothly decreases from the maximum value in a symmetrical manner; the broadening of the curve about the peak is proportional to the standard deviation σ. Wear-out-failures which almost suddenly outbreak after a certain period approximate the normal distribution.

Let us first examine a simple binomial distribution model to know concretely what the normal distribution expresses.

In the Bernoulli trial, the result of observation is either ① $+\sigma/\sqrt{N}$ or ② $-\sigma/\sqrt{N}$. The probability of ① or ② occurring is equally $\frac{1}{2}$. The initial value for starting the trial is 0 and we perform N tests. We obtain n times of ① and N - n times of ② , then the value x after N tests is

$$x = \frac{\sigma(2n - N)}{\sqrt{N}} \tag{IX-71}$$

This type of binomial distribution model approaches the normal distribution as we make the number of tests N sufficiently large.

$$f_{Bin}(n) = \frac{N!\left(\frac{1}{2}\right)^N}{n!(N-n)!} = \frac{N!\left(\frac{1}{2}\right)^N}{\left(\frac{1}{2}N + \frac{x}{2\sigma}\sqrt{N}\right)! \times \left(\frac{1}{2}N - \frac{x}{2\sigma}\sqrt{N}\right)!} \tag{IX-72}$$

If N is sufficiently large we can consider that x is a continuous variable. Hence

$$f_{Bin}(n)dn \rightarrow \psi(x)dx, \qquad dn \rightarrow \frac{\sqrt{N}}{2\sigma}dx \quad (N \rightarrow \infty) \tag{IX-73}$$

Here, $\psi(x)$ is given by

$$\psi(x) = \lim_{N\to\infty} \frac{\sqrt{N}}{2\sigma} f_{Bin}(n) = \lim_{N\to\infty} \left\{ \frac{\frac{\sqrt{N}}{2\sigma} \times N! \times \left(\frac{1}{2}\right)^N}{\left(\frac{1}{2}N + \frac{x}{2\sigma}\sqrt{N}\right)! \times \left(\frac{1}{2}N - \frac{x}{2\sigma}\sqrt{N}\right)!} \right\} \tag{IX-74}$$

Using Sterling's formula we can write

$$N! \approx \sqrt{2\pi N}\, N^N e^{-N} \qquad N > 10$$

$$\therefore \psi(x) = \lim_{N\to\infty} \left\{ \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{\sqrt{1 - \frac{x^2}{\sigma^2 N}}} \left(1 + \frac{x}{\sigma\sqrt{N}}\right)^{-\frac{N}{2} - \frac{x}{2\sigma}\sqrt{N}} \times \left(1 - \frac{x}{\sigma\sqrt{N}}\right)^{-\frac{N}{2} + \frac{x}{2\sigma}\sqrt{N}} \right\}$$

$$= \lim_{N\to\infty} \left\{ \frac{1}{\sigma\sqrt{2\pi}} \left(1 - \frac{x^2}{\sigma^2 N}\right)^{-\frac{N}{2} - \frac{1}{2}} \times \left(1 + \frac{x}{\sigma\sqrt{N}}\right)^{\frac{x}{2\sigma}\sqrt{N}} \times \left(1 - \frac{x}{\sigma\sqrt{N}}\right)^{-\frac{x}{2\sigma}\sqrt{N}} \right\} \tag{IX-75}$$

Here, using the exponential function

$$e^Z = \lim_{N\to\infty}\left(1 + \frac{Z}{n}\right)^n$$

We obtain

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{IX-76}$$

ψ(x) is known as the standard normal distribution. The general form of the normal probability density function is

$$f_{Norm}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad \text{(IX-77)}$$

By applying Eq. IX-77 to Eq. IX-8, we can calculate the mean value and the variance, which, we can confirm, will be μ and σ^2. The variable y used to convert f_{Norm} (X) to Eq. IX-76 is

$$y = \frac{x-\mu}{\sigma} \qquad \text{(IX-78)}$$

This is known as the standard normal variable. Figure IX-15 shows the typical characteristics of f_{Norm} (x) and ψ (y). In Fig. IX-15, the following calculations can be made.

$$\int_{\mu-\sigma}^{\mu+\sigma} f_{Norm}(x)dx \fallingdotseq 0.6826, \quad \int_{\mu-2\sigma}^{\mu+2\sigma} f_{Norm}(x)dx \fallingdotseq 0.9545, \quad \int_{\mu-3\sigma}^{\mu+3\sigma} f_{Norm}(x)dx \fallingdotseq 0.9973$$

When the value lnX rather than X behaves according to the normal distribution, we have a logarithmic normal distribution. To serve as an example, consider the life L in Eq. IX-33. Let us assume that a given lot of semiconductor devices are storage tested at temperature T. As long as there is no great variation in the quality of this lot, the distribution of the life L as caused by the stress placed on the devices by the temperature T is expressed as a logarithmic normal distribution and should be analyzed as such. Moreover, the logarithmic normal distribution is also used for the analysis of oxide film life by TDDB

$$f_{log-Norm}(X) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left(-\frac{(\ln X - \ln X_0)^2}{2\sigma^2}\right) \quad X>0 \qquad \text{(IX-79)}$$

Mean value $= e^{\ln X_0 + \frac{\sigma^2}{2}}$, Variance $= e^{2\ln X_0 + \sigma^2} \times (e^{\sigma^2} - 1)$

where X_0 is the median value of the probability distribution: (Eq.) : $\int_0^{X_0} f_{log-Norm}(X)dX = \int_{X_0}^{\infty} f_{log-Norm}(X)dX$

σ^2 is the variance of the normal distribution.

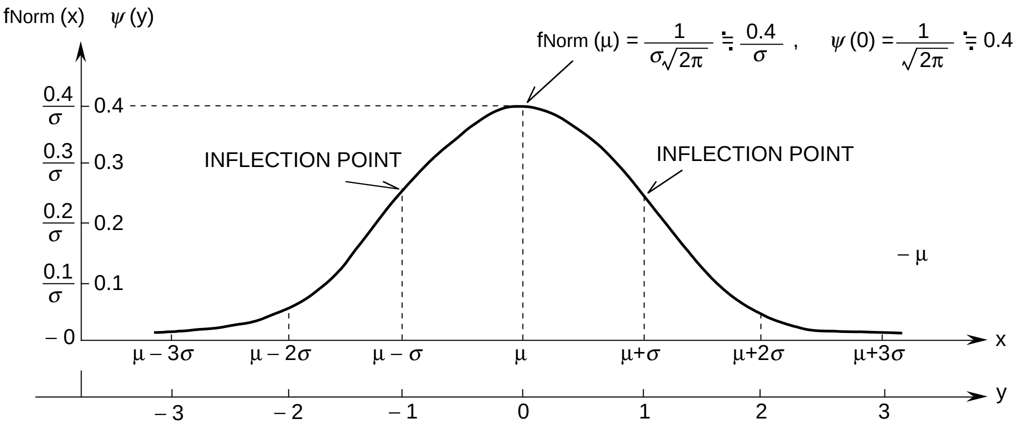


Fig. IX-15 Probability Density Function of Normal Distribution

2.4.11 WEIBULL DISTRIBUTION $f_{Wbl}(t)$

A set of n semiconductor devices is subjected to operation life test all together. As time t passes, the failure rate of these semiconductor devices changes. For studying the change, in this model, probabilities of individual devices failing by the time t are uniformly p(t), and the probability that at least one of the n devices will fail is F(t). Reversely the probability that none of the n devices will fail by the time t is R(t), which is, from Eq. IX-4

$$R(t)=1-F(t)=\{1-p(t)\}^{n} \quad \text{(IX-80)}$$

The Weibull distribution characteristically assumes that the n devices are as a lot expressed by the following reliability function.

$$R(t)=\{1-p(t)\}^{n}=e^{-\phi(t)} \quad \text{(IX-81)}$$

Let us make arrangements to express properly the failure distribution trends that we know through experience in the function form of Eq. IX-81.

We know through experience that no failures occur before a given test time γ and after γ the total number of failed devices increases with time t (or more correctly, maintains a non-decreasing trend). To express this empirically observed fact, $\phi(t)$ should have the characteristics below.

$$\phi(t)=0 \quad (0<t\le\gamma)$$

$$\phi(t)\ge 0, \quad \frac{d}{dt}\phi(t)\ge 0 \quad (\gamma<t)$$

Hence we choose the following form of function.

$$\left.\begin{aligned} &\phi(t)=0 \quad (0<t\le\gamma) \\ &\phi(t)=\frac{(t-\gamma)^{m}}{t_0} \quad (\gamma<t) \end{aligned}\right\} \quad \text{(IX-82)}$$

Therefore A

$$F_{Wbl}(t)=1-\{1-p(t)\}^{n}=1-e^{-\phi(t)}=1-\exp\left\{-\frac{(t-\gamma)^{m}}{t_0}\right\} \quad \text{(IX-83)}$$

Equation IX-83 is referred to as the Weibull failure distribution function.

Equation IX-83 of the Weibull distribution has three parameters, m, γ, and t_0, which are the shape parameter, position parameter, and scale parameter, respectively. The position parameter $\gamma=0$ if we assume that the probability of failure is already above 0 immediately before testing. From Eq. IX-83 and according to Eqs. IX-3 and IX-8

$$\left.\begin{aligned} &F_{Wbl}(t)=\frac{m(t-\gamma)^{m-1}}{t_0}\exp\left\{-\frac{(t-\gamma)^{m}}{t_0}\right\} \\ &\mu=t_0^{\frac{1}{m}}\,\Gamma\left(1+\frac{1}{m}\right), \quad \sigma^2=t_0^{\frac{2}{m}}\left\{\Gamma\left(1+\frac{2}{m}\right)-\Gamma^2\left(1+\frac{1}{m}\right)\right\} \end{aligned}\right\} \quad \text{(IX-84)}$$

$$\lambda_{Wbl}(t)=\frac{m}{t_0}(t-\gamma)^{m-1}$$

If m = 1, $\lambda_{Wbl}=1/t_0$ = constant, so the distribution is exponential.

If m > 1, $\lambda_{Wbl}(t)$ monotonically increases, representing a wear-out failure mode.

If m < 1, $\lambda_{Wbl}(t)$ monotonically decreases, representing an initial failure mode.

The function form of the Weibull distribution is capable of representing different failure modes depending on the value of the parameter m. Figure IX-16 shows how the form of $f_{Wbl}(t)$ changes with various values of m under conditions of $\gamma = 0$ and $t_0 = 1$.

When the position parameter $\gamma = 0$, Eq. IX-80 of the Weibull reliability function becomes

$$R_{Wbl}(t) = \int_t^{\infty} f_{Wbl}(t)dt = \int_t^{\infty} \frac{m}{t_0}\, t^{m-1} e^{-\frac{t^m}{t_0}}\, dt = e^{-\frac{t^m}{t_0}} \qquad \text{(IX-85)}$$

If we take the natural logarithm of Eq. IX-85 twice we obtain

$$\ln\ln \frac{1}{R_{Wbl}(t)} = \ln\ln \frac{1}{1 - F_{Wbl}(t)} = m \ln t - \ln t_0 \qquad \text{(IX-86)}$$

Here we rewrite as $\ln\ln \frac{1}{1 - F_{Wbl}(t)} = Y$, $\ln t = X$, $\ln t_0 = h$

Then Eq. IX-86 becomes

$$Y = mX - h \qquad \text{(IX-87)}$$

The Weibull chart is a chart in which the value of $F_{Wbl}(t)$ being converted into the length $\ln\ln \frac{1}{1 - F_{Wbl}(t)}$ is plotted on the vertical axis and time t is plotted on the horizontal axis on a logarithmic scale. The cumulative failure rate $F_{Wbl}(t)$ determined from observed data is described by the Weibull distribution. It is represented by a straight line according to the relation expressed by Eq. IX-87. The Weibull chart is useful for analysis of failure mode since observed data are graphed on the Weibull chart in the shape of an linear expression.

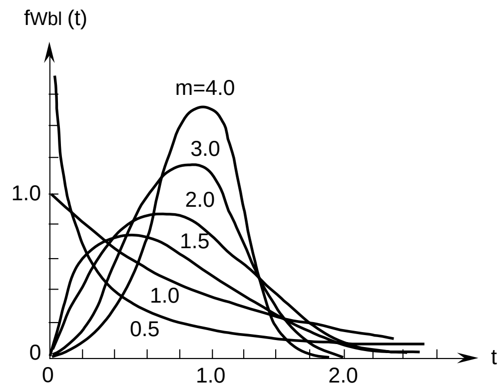


Fig IX-16 $f_{Wbl}(t) = mt^{m-1}e^{-t^m}$

2.4.12 DOUBLE EXPONENTIAL DISTRIBUTION $f_{d\text{-}exp}(x)$

If $\gamma = 0$ in Eq. IX-84 of the Weibull distribution

$$f_{Wbl}(t) = \frac{m}{t_0}\, t^{m-1}\, e^{-\frac{t^m}{t_0}} \qquad \text{(IX-88)}$$

in which, if we perform logarithmic transformation of t so that $x = \ln t$ and $t = e^X$, we have

$$f_{Wbl}(t)dt = \frac{m}{t_0}\, e^{mx}\ \text{¥}\ e^{-\frac{e^{mx}}{t_0}}\, dx = me^{mx - \ln t_0}\ \text{¥}\ \exp\left(-e^{(mx - \ln t_0)}\right) dx \qquad \text{(IX-89)}$$

This is the logarithmic transformation of the original distribution. Here we restate

$m = \lambda$, $\ln t_0 = \alpha$

$$f_{d-exp}(x) = \lambda e^{\lambda x - \alpha} ¥ e^{-e^{\lambda x - \alpha}} \qquad \text{(IX-90)}$$

Equation IX-90 is known as the double exponential (or extreme value) distribution. Here λ and σ are the scale and position parameters, respectively.

$$\mu = \frac{\alpha - \gamma}{\lambda}, \quad \sigma^2 = \frac{\varepsilon^2}{\lambda^2}$$

where γ = Euler's constant = 0.577....., $\varepsilon = \frac{\pi}{\sqrt{6}}$ = 1.283.....

When performing reliability tests on devices and subjecting them to stress, the damage incurred by all parts of the device is not necessarily equal. The part of the device which is most susceptible to the applied stress will be the most damaged and will eventually fail. It becomes the determining factor in the life of the device. This occurs in the case of a surge pulse withstand or mechanical impact test. In such a case, stress is applied locally to the device and the life or the withstanding limit of the device depends on its weak point. The double exponential distribution is suitable for analysis of such kinds of phenomena. To simplify Eq. IX-90 we use

$$-y = \lambda x - \alpha \qquad \text{(IX-91)}$$

to yield

$$f_{d-exp}(y) = \lambda e^{-y} e^{-e^{-y}} \qquad \text{(IX-92)}$$

$$F_{d-exp}(y) = e^{-e^{-y}} \qquad \text{(IX-93)}$$

If we take the natural logarithm of this twice, we have

$$\ln \ln \frac{1}{F_{d-exp}(y)} = y = \lambda x - \alpha \qquad \text{(IX-94)}$$

As seen by Eq. IX-87 related to the Weibull chart, the observed data can be plotted on extremal probability paper in the shape of a linear expression derived from Eq. IX-94. Thus it is possible to determine the scale parameter λ and position parameter σ.

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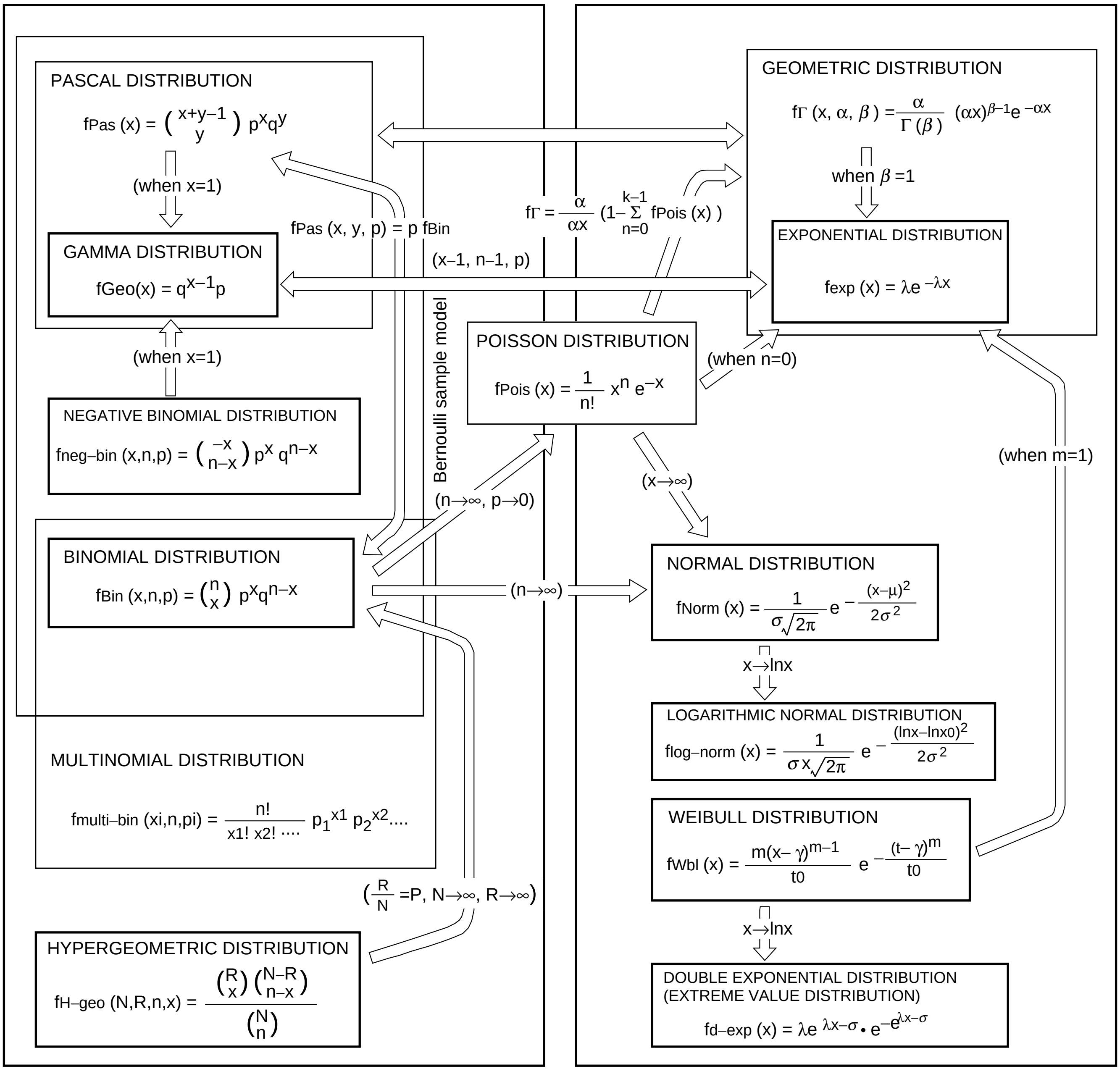
3. Japan Industrial Standards: Quality Control, New Edition, 1977 (In Japanese)

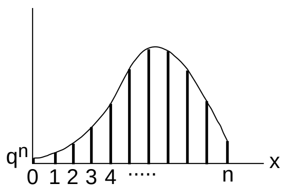
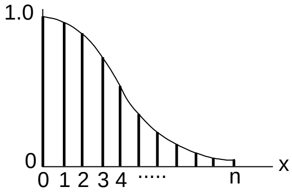
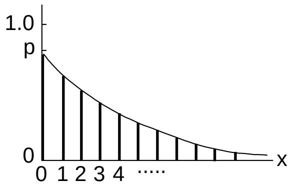
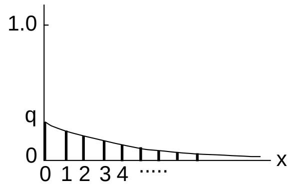
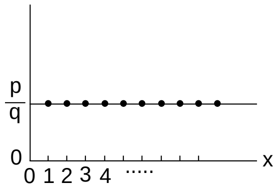
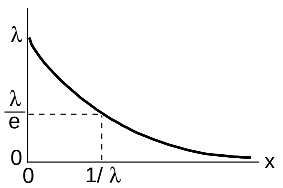
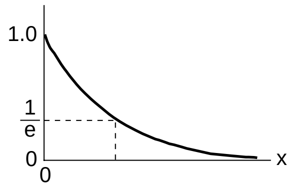
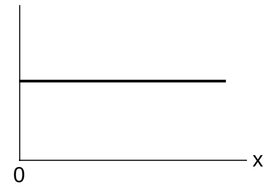
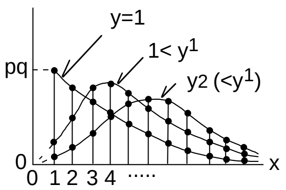
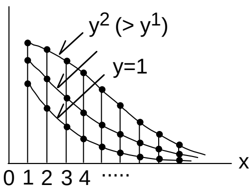
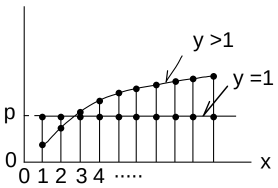
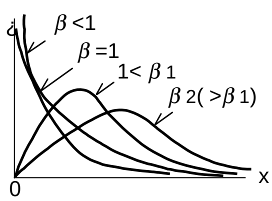
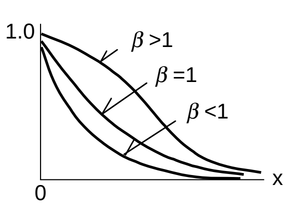
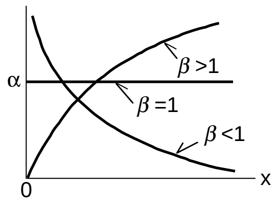
RELATIONS OF PROBABILITY DISTRIBUTIONS

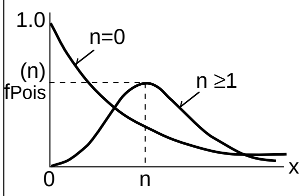
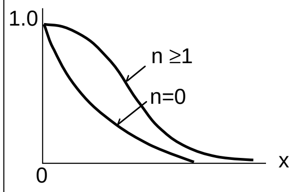
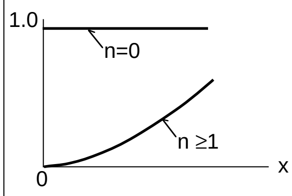
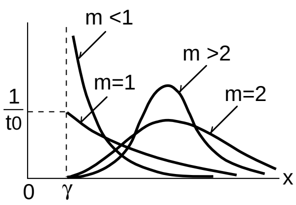
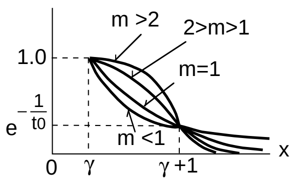
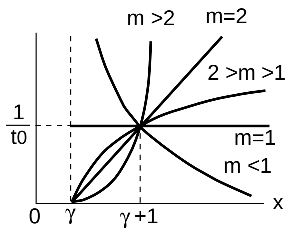
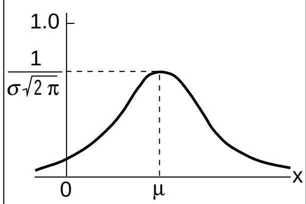
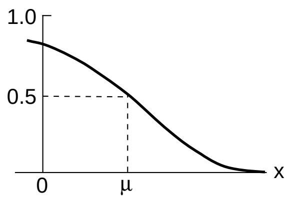
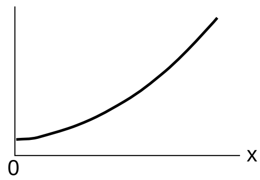
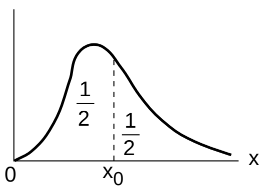
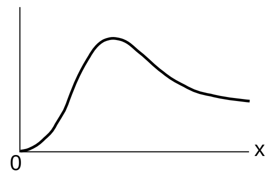


(DISCRETE DISTRIBUTION MODEL)

(CONTINUOUS DISTRIBUTION MODEL)

PROBABILITY FUNCTIONS

Probability Distribution	Probability density function f(x)	Reliability function R(x)	Failure Rate λ (x)	Mean Value μ	Variance σ^2	Remark
Binomial distribution	$f_{Bin}(x,n,p) = \binom{n}{x} p^x q^{n-x}$ 	$R_{Bin}(x) = \sum_{i=x+1}^{n} \binom{n}{i} p^i q^{n-i}$ 		np	npq	
Geometric distribution	$f_{Geo}(x) = q^x p$ 	$R_{Geo}(x) = q^{x+1}$ 	$\lambda_{Geo} = \frac{p}{q}$ = const 	$\frac{q}{p}$	$\frac{q}{p^2}$	
Exponential distribution	$f_{exp}(x) = \lambda e^{-\lambda x}$ 	$R_{exp}(x) = e^{-\lambda x}$ 	$\lambda_{exp} = \lambda$ =const 	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	
Pascal distribution	$f_{Pas}(x) = \left(\frac{x+y-1}{y}\right) p^x q^y$ 	$R_{Pas}(x) = \sum_{i=x}^{n-1} \binom{n-1}{i} p^{n-i} q^i$ 	$\lambda_{Pas} = \frac{f_{Pas}}{R_{Pas}}$ 	$\frac{qy}{p}$	$\frac{qy}{p^2}$	
Gamma distribution	(x, α, β) 	$R_\Gamma(x) = \frac{\alpha}{\Gamma(\beta)} \int_x^\infty (a\,t)^{b-1} e^{-\alpha t}\,dt$ 	$\lambda_\Gamma = \frac{f_\Gamma}{R_\Gamma}$ 	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha^2}$	α : scale parameter β : shape parameter Refer to Fig. IX-13

Probability Distribution	Probability density function f(x)	Reliability function R(x)	Failure Rate λ (x)	Mean Value μ	Variance σ^2	Remark
Poisson distribution	$f_{Pois}(x) = \frac{1}{n!} x^n e^{-x}$ 	$R_{Pois}(x) = \frac{1}{n!} \int_x^\infty t^n e^{-t} dt$ 	$\lambda_{Pois} = \frac{f_{Pois}}{R_{Pois}}$ 	X	X^2	Refer to Fig. IX-14
Weibull distribution	$f_{Wbl}(x) = \frac{m(x-\gamma)^{m-1}}{t_0} \bullet e^{-\frac{(x-\gamma)^m}{t_0}}$ 	$R_{Wbl}(x) = e^{-\frac{(x-\gamma)^m}{t_0}}$ 	$\lambda_{Wbl}(x) = \frac{m}{t_0}(x-\gamma)^{m-1}$ 	$t_0^{\frac{1}{m}}\Gamma\left(1+\frac{1}{m}\right)$	$t_0^{\frac{2}{m}}\left\{\Gamma\left(1+\frac{2}{m}\right) - \Gamma^2\left(1+\frac{1}{m}\right)\right\}$	m: shape parameter γ : position parameter t0: scale parameter Refer to Fig. IX-16
Normal distribution	$f_{Norm}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ 	$R_{Norm}(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_x^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}} dt$ 	$\lambda_{Norm} = \frac{f_{Norm}}{R_{Norm}}$ 	μ	σ^2	Refer to Fig. IX-15
Logarithmic normal distribution	$f_{log-norm}(x) = \frac{1}{\sigma x \sqrt{2\pi}} e^{-\frac{(\ln x - \ln X_0)^2}{2\sigma^2}}$ 	$R_{log-norm}(x) = \int_x^\infty f_{log\text{-}norm}(t)\,dt$ 	$\lambda_{log-norm} = \frac{f_{l-norm}}{R_{l-norm}}$ 	$e^{-\left(\ln x_0+\frac{\sigma^2}{2}\right)}$	$e^{2\ln x_0+\sigma^2} \times (e^{\sigma^2}-1)$	σ^2: variance of normal distribution X0: median value of probability distribution